

Dynamical SUSY breaking in meta-stable vacua

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ABSTRACT: Dynamical supersymmetry breaking in a long-lived meta-stable vacuum is a phenomenologically viable possibility. This relatively unexplored avenue leads to many new models of dynamical supersymmetry breaking. Here, we present a surprisingly simple class of models with meta-stable dynamical supersymmetry breaking: $\mathcal{N} = 1$ supersymmetric QCD, with massive flavors. Though these theories are strongly coupled, we definitively demonstrate the existence of meta-stable vacua by using the free-magnetic dual. Model building challenges, such as large flavor symmetries and the absence of an R-symmetry, are easily accommodated in these theories. Their simplicity also suggests that broken supersymmetry is generic in supersymmetric field theory and in the landscape of string vacua.

KEYWORDS: Supersymmetry and Duality, Supersymmetry Breaking, Duality in Gauge Field Theories.

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1. Introduction

1.1 General remarks

At first glance, dynamical supersymmetry breaking appears to be a rather non-generic phenomenon in supersymmetric gauge theory. The non-zero Witten index of $\mathcal{N} = 1$ Yang-Mills theory immediately implies that any $\mathcal{N} = 1$ supersymmetric gauge theory with massive, vector-like matter has supersymmetric vacua [1]. So theories with no supersymmetric vacua must either be chiral, as in the original examples of [2, 3], or if they are non-chiral, they must have massless matter, as in the examples of [4, 5]. The known theories that satisfy these requirements and dynamically break supersymmetry look rather complicated, and applications to realistic model building only compounds the complications. The result has been a literature of rather baroque models of dynamical supersymmetry breaking and mediation. For reviews and references, see e.g. [6].

We point out that new model building avenues are opened up by abandoning the prejudice that models of dynamical supersymmetry breaking must have *no* supersymmetric vacua. This prejudice is unnecessary, because it is a phenomenologically viable possibility that we happen to reside in a very long lived, false vacuum, and that there is a supersymmetric vacuum elsewhere in field space. Meta-stable supersymmetry breaking vacua have been encountered before in the literature of models of supersymmetry breaking and mediation; some examples are [7–9]. Indeed, even if the supersymmetry breaking sector has no supersymmetric vacua, there is a danger that the mediation sector will introduce supersymmetric vacua elsewhere. Such encounters of meta-stable supersymmetry breaking are generally accompanied with a (justified) apology for the aesthetic defect and, in favorable cases, it is shown that the lifetime can nevertheless be longer than the age of the Universe.

The novelty here is that we accept meta-stable vacua from the outset, even in the supersymmetry breaking sector. This approach leads us immediately to many new and much simpler models of supersymmetry breaking. Classic constraints, needed for having *no* supersymmetric vacua, no longer constrain models of meta-stable supersymmetry breaking. For instance, theories with non-zero Witten index and/or with no conserved $U(1)_R$ symmetry [3, 10] can nevertheless have meta-stable supersymmetry breaking vacua. A condition for supersymmetry breaking that does still apply in the meta-stable context is the need for a massless fermion to play the role of the Goldstino. But even this condition can be subtle: the massless fermion can be present in the low-energy macroscopic theory, even if it is not obvious in the original, ultraviolet, microscopic theory. This happens in our examples.

Phenomenologically, we would like the lifetime of our meta-stable state to be longer than the age of the Universe. Moreover, the notion of meta-stable states is meaningful only when they are parametrically long lived. It is therefore important for us to have a dimensionless parameter, ϵ , whose parametric smallness guarantees the longevity of the meta-stable state. In our examples, ϵ is given by a ratio of a mass and a dynamical scale,

$$\epsilon \equiv \frac{\mu}{\Lambda_m} \sim \sqrt{\frac{m}{\Lambda}}, \quad (1.1)$$

where the masses and scales will be explained shortly. What happens to the meta-stable state as $\epsilon \rightarrow 0$ depends on what we hold fixed. In some examples, we should hold the dynamical scale Λ fixed, then as $\epsilon \rightarrow 0$, the meta-stable state becomes supersymmetric. In other examples, we should hold the mass scale μ fixed, and then supersymmetry is broken.

Most of the analysis of supersymmetry dynamics in the past has been concerned with BPS / chiral / holomorphic quantities, which are protected in some way by supersymmetry. Since we are interested in supersymmetry breaking, we have to go outside this domain, and our answers depend on non-chiral information which in general cannot be computed. In the past, calculable models of dynamical supersymmetry breaking were based on the fact that the vacuum ended up being at large fields, where the Kähler potential is approximately classical for the fields of the microscopic theory [3]. In this paper we study vacua at small field expectation values, where the Kähler potential is complicated. Here our small parameter ϵ will be useful. Taking $\epsilon \rightarrow 0$, holding fixed the dynamical scale Λ , supersymmetry is unbroken and we know the spectrum of the IR theory. When this theory is IR free, the Kähler metric of the light modes is smooth and it can be parameterized by a small number of real coefficients of order one. Even though we do not know how to compute these coefficients, we will be able to express a lot of information (the ground state energy, the spectrum of light particles, the effective potential, etc.) in terms of them. This approach has already been used in [11], to analyze the supersymmetry breaking model of [4, 5] in the strong coupling region.

1.2 Our main example

Our main example of meta-stable dynamical supersymmetry breaking in this paper is surprisingly simple: $\mathcal{N} = 1$ supersymmetric $SU(N_c)$ QCD, with N_f massive fundamental flavors. In order to have control over the theory in the IR, we take N_f in the free magnetic range [12–14], $N_c + 1 \leq N_f < \frac{3}{2}N_c$. We will show that, in addition to the expected supersymmetric vacua of a theory with massive vector-like matter, there are *long-lived non-supersymmetric vacua*. Our analysis is reliable in a particular limit,

$$|\epsilon| \sim \sqrt{\left|\frac{m}{\Lambda}\right|} \ll 1. \tag{1.2}$$

where m is the typical scale of the quark masses and Λ is the strong-coupling scale of the theory. Using the free-magnetic dual description of the theory in the infrared, we determine properties of the strongly coupled gauge theory outside of the usual realm of holomorphic quantities and supersymmetric vacua. The simplicity of these models leads us to suspect that meta-stable vacua with broken supersymmetry are generic.

In the infrared description of the theory, supersymmetry is spontaneously broken at tree-level by what we refer to as the “rank-condition” mechanism of supersymmetry breaking. Consider a theory of chiral superfields Φ_{ij} , φ_c^i , and $\tilde{\varphi}^{ic}$, with $i = 1 \dots N_f$, and $c = 1 \dots N$, with $N < N_f$, and tree-level superpotential

$$W = h \text{Tr } \varphi \Phi \tilde{\varphi} - h\mu^2 \text{Tr } \Phi. \tag{1.3}$$

The F-terms of Φ , $F_{\Phi_{ij}} \sim \tilde{\varphi}^{jc} \varphi_c^i - h\mu^2 \delta^{ij}$, cannot all vanish, because δ^{ij} has rank N_f but

$\tilde{\varphi}^{jc} \varphi_c^i$ only has rank $N < N_f$. Supersymmetry is thus spontaneously broken. For $SU(N_c)$ SQCD with $N_c + 1 \leq N_f < \frac{3}{2}N_c$, (1.3) arises as the infrared free, low-energy effective theory of the magnetic dual [13], with $N = N_f - N_c$ and $\mu \sim \sqrt{m\Lambda}$.

At tree-level in the macroscopic theory (1.3), there is a moduli space of degenerate, non-supersymmetric vacua, labelled by arbitrary expectation values of some classically massless fields, which are some components of the fields in (1.3). Some of these fields are Goldstone bosons of broken global symmetries, and remain as exactly massless moduli of the vacua. (The moduli space, being of the form G/H , is always compact.) There are also classically massless “pseudo-moduli”; these get a potential from perturbative quantum corrections in the effective theory (1.3). The leading perturbative contribution to the potential for the pseudo-moduli can be computed using the one-loop correction to the vacuum energy,

$$V_{\text{eff}}^{(1)} = \frac{1}{64\pi^2} \text{STr} \mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} \equiv \frac{1}{64\pi^2} \left(\text{Tr} m_B^4 \log \frac{m_B^2}{\Lambda^2} - \text{Tr} m_F^4 \log \frac{m_F^2}{\Lambda^2} \right), \quad (1.4)$$

where m_B^2 and m_F^2 are the tree-level boson and fermion masses, as a function of the expectation values of the pseudo-moduli.¹ Using (1.4), we find non-supersymmetric vacua, stabilized by a potential barrier which to leading order scales like $|\mu^2|$. In terms of the parameter μ^2 appearing in (1.3), this effective potential is thus not real analytic. That is why this potential, computed in the low-energy macroscopic theory, is robust upon including effects from the underlying microscopic theory. We will discuss this in more detail below.

The N_c supersymmetric vacua expected from the Witten index of $SU(N_c)$ SQCD with massive matter can also be seen in the low-energy macroscopic theory of the free magnetic dual. Giving the fields Φ in (1.3) expectation values, gaugino condensation in the $SU(N)$ magnetic dual contributes to the superpotential and leads to the expected N_c supersymmetric vacua. This is an interesting example of non-perturbative *restoration* of supersymmetry in a theory which breaks supersymmetry at tree-level.

1.3 Outline

As we have summarized, our microscopic UV theory is $SU(N_c)$ SQCD, and we analyze its supersymmetry-breaking dynamics using the macroscopic, IR-free dual. In the body of the paper, we will follow a bottom-up presentation, starting in the IR, and then working up to the UV. The advantage of this bottom up approach is that, as we shall discuss, the important physics of supersymmetry breaking all happens in the infrared theory. Effects from the underlying, microscopic theory do not significantly affect the conclusions. These considerations apply more broadly than to the particular models that we analyze here.

In section 2, we discuss the rank-condition supersymmetry breaking in the macroscopic, low-energy theory (1.3), taking $SU(N)$ to be a global, rather than gauge symmetry. We compute the leading effect from the one-loop potential (1.4). These theories have absolutely

¹The ultraviolet cutoff Λ in (1.4) can be absorbed into the renormalization of the coupling constants appearing in the tree-level vacuum energy V_0 . In particular, $\text{STr} \mathcal{M}^4$ is independent of the pseudo-moduli.

stable, non-supersymmetric vacua. In section 3, we gauge the $SU(N)$ group, taking $N_f > 3N$ (which becomes $N_f < \frac{3}{2}N_c$ in the electric theory, after using $N = N_f - N_c$) so the theory is infrared free. The $SU(N)$ gauge group is completely Higgsed in the non-supersymmetric vacua, and the leading quantum effective potential is essentially the same as that found in section 2. The $SU(N)$ gauge fields do not much affect the non-supersymmetric vacua, but they do have an important effect elsewhere in field space, where they lead to non-perturbative restoration of supersymmetry. So the non-supersymmetric vacua are only meta-stable, once $SU(N)$ is gauged.

In section 4, we provide a short, general discussion on why it is valid to take a bottom up approach, analyzing supersymmetry breaking and the vacuum in the low-energy, macroscopic effective theory. It is argued in general that effects from the underlying microscopic theory, whatever they happen to be, do not significantly affect the conclusions.

In section 5, we connect the macroscopic effective field theories, studied in the previous sections, with a microscopic description in terms of $SU(N_c)$ SQCD with N_f fundamental flavors. The fields Φ_{ij} and φ^i and $\tilde{\varphi}^j$ are composite objects of the microscopic theory. As discussed in section 4, strong quantum effects of the underlying microscopic theory do not alter our conclusions about the meta-stable supersymmetry breaking vacuum.

In section 6, we discuss analogous models of meta-stable supersymmetry breaking, based on $SO(N)$ (or more precisely, $Spin(N)$) and $Sp(N)$ groups with fundamental matter. For the case of $Spin(N)$, we argue that the meta-stable non-supersymmetric vacua and the supersymmetric vacua are in different phases: one is confining, and the other is oblique confining.

In section 7, we show that our meta-stable vacua can be made parametrically long lived. This makes them well defined and phenomenologically interesting. Finally, in section 8, we make some preliminary comments about applications to model building.

In appendix A, we review some basic aspects of F-term supersymmetry breaking. In appendix B, we provide some technical details of the computation of the one-loop effective potential in section 2. In appendix C, we present supersymmetric gauge theories, based $SU(N)$ supersymmetric gauge theory with adjoint matter, which have landscapes of supersymmetry breaking vacua. Such gauge theories can naturally arise in string theory. In appendix D, we suggest testing for meta-stable non-supersymmetric vacua in the context of $\mathcal{N} = 2$ supersymmetry, with small explicit breaking to $\mathcal{N} = 1$, using the exactly known $\mathcal{N} = 2$ Kähler potential of [15] and following works. For the particular case of $SU(2)$ with no matter, we observe that there is no meta-stable, non-supersymmetric vacuum.

2. The macroscopic model: part I

In this section we discuss our macroscopic theory (1.3) without the gauge interactions. This is a Wess-Zumino model with global symmetry group

$$SU(N) \times SU(N_f)^2 \times U(1)_B \times U(1)' \times U(1)_R \tag{2.1}$$

(later we will identify $N = N_f - N_c$), with $N_f > N$ and the following matter content

	SU(N)	SU(N _f)	SU(N _f)	U(1) _B	U(1)'	U(1) _R	
Φ	1	□	□̄	0	-2	2	(2.2)
φ	□	□̄	1	1	1	0	
φ̃	□̄	1	□	-1	1	0	

We will take the canonical Kähler potential,

$$K = \text{Tr } \varphi^\dagger \varphi + \text{Tr } \tilde{\varphi}^\dagger \tilde{\varphi} + \text{Tr } \Phi^\dagger \Phi \tag{2.3}$$

and tree-level superpotential

$$W = h \text{Tr } \varphi \Phi \tilde{\varphi} - h \mu^2 \text{Tr } \Phi. \tag{2.4}$$

The first term in (2.4) is the most general W_{tree} consistent with the global symmetries (2.1). The second term in (2.4) breaks the global symmetry to $SU(N) \times SU(N_f) \times U(1)_B \times U(1)_R$, where the unbroken $SU(N_f)$ is the diagonal subgroup of the original $SU(N_f)^2$.

Since $N_f > N$, the F-terms cannot be simultaneously set to zero, and so supersymmetry is spontaneously broken by the rank condition, as described in the introduction. The scalar potential is minimized, with

$$V_{min} = (N_f - N) |h^2 \mu^4|, \tag{2.5}$$

along a classical moduli space of vacua which, up to global symmetries, is given by

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \end{pmatrix}, \quad \varphi = \begin{pmatrix} \varphi_0 \\ 0 \end{pmatrix}, \quad \tilde{\varphi}^T = \begin{pmatrix} \tilde{\varphi}_0 \\ 0 \end{pmatrix}, \quad \text{with } \tilde{\varphi}_0 \varphi_0 = \mu^2 \mathbb{1}_N. \tag{2.6}$$

Here Φ_0 is an arbitrary $(N_f - N) \times (N_f - N)$ matrix, and φ_0 and $\tilde{\varphi}_0$ are $N \times N$ matrices (the zero entries in (2.6) are matrices).

The vacua of maximal unbroken global symmetry are (up to unbroken flavor rotations)

$$\Phi_0 = 0, \quad \varphi_0 = \tilde{\varphi}_0 = \mu \mathbb{1}_N, \tag{2.7}$$

This preserves an unbroken $SU(N)_D \times SU(N_f - N) \times U(1)_{B'} \times U(1)_R$, as well as a discrete charge conjugation symmetry that exchanges φ and $\tilde{\varphi}$.

We now examine the one-loop effective potential of the classical pseudo-flat directions around the vacua (2.7). To simplify the presentation, we will expand around (2.7) and show that the classical pseudo-moduli there get positive mass-squared.

To see what the light fields are, we expand around (2.7) using the parametrization

$$\Phi = \begin{pmatrix} \delta Y & \delta Z^T \\ \delta \tilde{Z} & \delta \hat{\Phi} \end{pmatrix}, \quad \varphi = \begin{pmatrix} \mu + \frac{1}{\sqrt{2}}(\delta\chi_+ + \delta\chi_-) \\ \frac{1}{\sqrt{2}}(\delta\rho_+ + \delta\rho_-) \end{pmatrix}, \quad \tilde{\varphi}^T = \begin{pmatrix} \mu + \frac{1}{\sqrt{2}}(\delta\chi_+ - \delta\chi_-) \\ \frac{1}{\sqrt{2}}(\delta\rho_+ - \delta\rho_-) \end{pmatrix} \tag{2.8}$$

(Here δY and $\delta\chi_{\pm}$ are $N \times N$ matrices, and δZ , $\delta\tilde{Z}$, and $\delta\rho_{\pm}$ are $(N_f - N) \times N$ matrices.) The potential from (2.4) gives most of the fields tree-level masses $\sim |h\mu|$. There are also massless scalars, some of which are Goldstone bosons of the broken global symmetries:

$$\frac{\mu^*}{|\mu|}\delta\chi_- - h.c., \quad \text{Re}\left(\frac{\mu^*}{|\mu|}\delta\rho_+\right), \quad \text{Im}\left(\frac{\mu^*}{|\mu|}\delta\rho_-\right). \quad (2.9)$$

The first is in $\text{SU}(N) \times \text{SU}(N)_F \times \text{U}(1)_B / \text{SU}(N)_D$, and the latter two are in $\text{SU}(N_f) / \text{SU}(N)_F \times \text{SU}(N_f - N) \times \text{U}(1)_{B'}$, where $\text{SU}(N)_F \subset \text{SU}(N_f)$.

The other classically massless scalars are fluctuations of the classical pseudo-flat directions,

$$\delta\hat{\Phi} \quad \text{and} \quad \delta\hat{\chi} \equiv \frac{\mu^*}{|\mu|}\delta\chi_- + h.c. \quad (2.10)$$

These pseudo-moduli acquire masses, starting at one-loop, from their couplings to the massive fields. The effective theory for the pseudo-moduli has the form

$$\mathcal{L}_{\text{eff}} = \text{Tr} \partial(\delta\hat{\Phi})^\dagger \partial(\delta\hat{\Phi}) + \frac{1}{2} \text{Tr} (\partial(\delta\hat{\chi}))^2 - V_{\text{eff}}^{(1)}(\delta\hat{\Phi}, \delta\hat{\chi}) + \dots \quad (2.11)$$

where \dots denotes higher order derivative interactions, as well as terms coming from two or more loops of the massive fields. The one-loop contribution to the effective potential dominates over higher loops, because the coupling h is (marginally) irrelevant in the infrared. The kinetic terms in (2.11) are inherited from the tree-level kinetic terms from (2.3) of the full theory, so they are diagonal and canonical to leading order.

The one-loop effective potential for the pseudo-moduli can be computed from the one-loop correction (1.4) to the vacuum energy, in the background where the pseudo-moduli have expectation values. Expanding to quadratic order around the vacua (2.7), the effective potential for the pseudo-moduli must be of the form

$$V_{\text{eff}}^{(1)} = |h^4\mu^2| \left(\frac{1}{2}a \text{Tr} \delta\hat{\chi}^2 + b \text{Tr} \delta\hat{\Phi}^\dagger \delta\hat{\Phi} \right) + \dots, \quad (2.12)$$

for some numerical coefficients a and b . Here we used the global symmetries and the fact that only single traces appear in (1.4)² to determine the field dependence in (2.12). The factor of $|h^4\mu^2|$ follows from dimensional analysis and the fact that the classical masses in \mathcal{M} are all proportional to h . Substituting the classical masses into (1.4), the result is

$$a = \frac{\log 4 - 1}{8\pi^2}(N_f - N), \quad b = \frac{\log 4 - 1}{8\pi^2}N. \quad (2.13)$$

Some details of the calculation of a and b are given in appendix B, where we also show how our macroscopic model is related to an O’Raifeartaigh-like model of supersymmetry breaking. In any event, the precise values of a and b are not too important; what matters for us is that they are both positive. The leading order effective potential for the pseudo-moduli is

$$V_{\text{eff}}^{(1)} = \frac{|h^4\mu^2|(\log 4 - 1)}{8\pi^2} \left(\frac{1}{2}(N_f - N) \text{Tr} \delta\hat{\chi}^2 + N \text{Tr} \delta\hat{\Phi}^\dagger \delta\hat{\Phi} \right) + \dots, \quad (2.14)$$

so the vacua (2.7) are indeed stable, without any tachyonic directions.

²Equivalently, it is easily verified that only planar diagrams contribute at one loop.

The spectrum of the theory in the vacuum (2.7) has a hierarchy of mass scales, dictated by the (marginally) irrelevant coupling h . Some fields have tree-level masses $\sim |h\mu|$. The pseudo-moduli have masses $\sim |h^2\mu|$ from (2.14). The Goldstone bosons of the broken global symmetries of course remain exactly massless; in particular, no quantum corrections could drive them tachyonic. There is also an exactly massless Goldstino, because supersymmetry is broken.

3. The macroscopic model: part II — dynamical SUSY restoration

We now gauge the $SU(N)$ symmetry of the previous section. We are interested in the case $N_f > 3N$, where the $SU(N)$ theory is IR free instead of asymptotically free. Thus the theory has a scale Λ_m , above which it is strongly coupled. (The subscript m on Λ_m is for “macroscopic.”) The running of the holomorphic gauge coupling of $SU(N)$ is given by

$$e^{-8\pi^2/g^2(E)+i\theta} = \left(\frac{E}{\Lambda_m}\right)^{N_f-3N}. \tag{3.1}$$

So g runs to zero in the infrared, and the theory there can be analyzed perturbatively. In the ultraviolet, we encounter a Landau pole at $E = |\Lambda_m|$; thus, for energies $E \sim |\Lambda_m|$ and above, the $SU(N)$ theory is not well defined. A different description of the theory is then needed.

Having gauged $SU(N)$, the scalar potential is now $V = V_F + V_D$, where V_F is the F -term potential discussed in the previous section, and V_D is the D -term potential

$$V_D = \frac{1}{2}g^2 \sum_A (\text{Tr } \varphi^\dagger T_A \varphi - \text{Tr } \tilde{\varphi} T_A \tilde{\varphi}^\dagger)^2. \tag{3.2}$$

The D -term potential (3.2) vanishes in the vacua (2.7), so (2.7) remains as a minimum of the tree-level potential. The $SU(N)$ gauge symmetry is completely Higgsed in this vacuum. Through the super-Higgs mechanism, the $SU(N)$ gauge fields acquire mass $g\mu$, the erstwhile Goldstone bosons $\text{Im}(\mu^* \delta\chi_- / |\mu|)'$ are eaten (the prime denotes the traceless part), and the erstwhile pseudo-moduli $\delta\tilde{\chi}' = \text{Re}(\mu^* \delta\chi_- / |\mu|)'$ get a non-tachyonic, tree-level mass $g\mu$ from (3.2).³ Thus, the fields $\delta\hat{\Phi}$ and $\text{Tr } \delta\hat{\chi}$ remain as classical pseudo-moduli.

We should compute the leading quantum effective potential for these pseudo-moduli, as in the previous section, to determine whether the vacua (2.7) are stabilized, or develop tachyonic directions. Actually, no new calculation is needed: the effect of the added $SU(N)$ gauge fields drops out in the leading order effective potential for the pseudo-moduli. The reason is that the tree-level spectrum of the massive $SU(N)$ vector supermultiplet is supersymmetric, so its additional contributions to the supertrace of (1.4) cancel. To see this, note that the $SU(N)$ gauge fields do not directly couple to the supersymmetry breaking: the D -terms (3.2) vanish on the pseudo-flat space, and the non-zero expectation values of φ and $\tilde{\varphi}$, which give the $SU(N)$ gauge fields their masses, do not couple directly to any non-zero F terms.

³We could have also gauged $U(N) \cong SU(N) \times U(1)_B$ in (2.2), giving $U(1)_B$ gauge coupling g' . Then the $U(1)_B$ vector multiplet gets a tree-level supersymmetric mass $\sim g'\mu$ in the vacuum (2.7), by the super Higgs mechanism. In particular, its trace part, $\text{Tr } \delta\hat{\chi}$ gets a non-tachyonic mass $\sim g'\mu$ at tree-level.

We conclude that the leading order effective potential (2.14) for the pseudo-moduli is unaffected by the gauging of $SU(N)$. The vacua are as in (2.7), with broken supersymmetry and no tachyonic directions.

Though gauging the $SU(N)$ does not much affect the supersymmetry breaking vacua (2.7), it does have an important effect elsewhere in field space: it leads to supersymmetric vacua. To see this, consider giving Φ general, non-zero expectation values. By the superpotential (2.4), this gives the $SU(N)$ fundamental flavors, φ and $\tilde{\varphi}$, mass $\langle h\Phi \rangle$. Below the energy scale $\langle h\Phi \rangle$, we can integrate out these massive flavors. The low-energy theory is then $SU(N)$ pure Yang-Mills, with holomorphic coupling given by

$$e^{-8\pi^2/g^2(E)+i\theta} = \left(\frac{\Lambda_L}{E}\right)^{3N} = \frac{h^{N_f} \det \Phi}{\Lambda_m^{N_f-3N} E^{3N}}. \quad (3.3)$$

In the last equality, we matched the running coupling to that above the energy scale $\langle h\Phi \rangle$, as given in (3.1). The low-energy theory has superpotential

$$W_{\text{low}} = N(h^{N_f} \Lambda_m^{-(N_f-3N)} \det \Phi)^{1/N} - h\mu^2 \text{Tr} \Phi, \quad (3.4)$$

where the first term comes from $SU(N)$ gaugino condensation, upon using (3.3) to relate Λ_L to Λ_m . We stress that the appearance of Λ_m in (3.4) does not signify that we are including any effects coming from physics at or above the ultraviolet cutoff Λ_m . Rather, it appears because we have expressed the infrared free coupling g as in (3.1).

Extremizing the superpotential (3.4), we find $N_f - N$ supersymmetric vacua at

$$\langle h\Phi \rangle = \Lambda_m \epsilon^{2N/(N_f-N)} \mathbb{1}_{N_f} = \mu \frac{1}{\epsilon^{(N_f-3N)/(N_f-N)}} \mathbb{1}_{N_f}, \quad \text{where} \quad \epsilon \equiv \frac{\mu}{\Lambda_m}. \quad (3.5)$$

Note that, for $|\epsilon| \ll 1$,

$$|\mu| \ll |\langle h\Phi \rangle| \ll |\Lambda_m|. \quad (3.6)$$

Because $\langle h\Phi \rangle$ is well below the Landau pole at Λ_m , this analysis in the low-energy, macroscopic theory is justified and reliable. As we will discuss in section 7, $|\mu| \ll |\langle h\Phi \rangle|$ also guarantees the longevity of the meta-stable, non-supersymmetric vacua (2.7).

We see here an amusing phenomenon: dynamical supersymmetry *restoration*, in a theory that breaks supersymmetry at tree-level. For $\Lambda_m \rightarrow \infty$ with μ fixed, the theory breaks supersymmetry. For Λ_m large but finite (corresponding to small but nonzero ϵ), a supersymmetric vacuum comes in from infinity. The relevant non-perturbative effect arises in an IR free gauge theory, and it can be reliably computed.

The existence of these supersymmetric vacua elsewhere in field space implies that the non-supersymmetric vacua of the previous section become only meta-stable upon gauging $SU(N)$. The model with gauged $SU(N)$ therefore exhibits meta-stable supersymmetry breaking. We shall realize it dynamically in section 5.

We note that our conclusions are in complete accord with the connection of [3, 10] between the existence of a $U(1)_R$ symmetry and broken supersymmetry. The theory of the previous section has a conserved $U(1)_R$ symmetry, and it has broken supersymmetry. In the theory of this section, there is no conserved $U(1)_R$ symmetry, because it is anomalous

under the gauged $SU(N)$; this breaking is explicit in (3.4). Correspondingly, there are supersymmetric vacua. For $\langle \Phi \rangle$ near the origin, the $SU(N)$ gauge theory is IR free, so the $U(1)_R$ symmetry returns as an accidental symmetry of the infrared theory. So supersymmetry breaking in our meta-stable vacuum near the origin is related to the accidental R-symmetry there.

4. Effects from the underlying microscopic theory

The theory we discussed in the previous sections is IR free and therefore it cannot be a complete theory. It breaks down at the UV scale $|\Lambda_m|$ where its gauge interactions become large. (The coupling h in (2.4) also has a Landau pole; for simplicity we discuss only a single scale $|\Lambda_m|$.) In this section we will examine whether our results above depend on the physics at the scale $|\Lambda_m|$ which we do not have under control. The only dimensionful parameter of the low energy theory is μ and therefore, we will assume

$$|\epsilon| = \left| \frac{\mu}{\Lambda_m} \right| \ll 1 \tag{4.1}$$

We will argue that the inequality (4.1) guarantees that our calculations above give the dominant effect in the low energy theory.

The first effect that we should worry about is loops of modes from the high energy theory. These can be summarized by correction terms in the effective Kähler potential, which at quartic order take the typical form

$$\delta K = \frac{c}{|\Lambda_m|^2} \text{Tr}(\Phi^\dagger \Phi)^2 + \dots, \tag{4.2}$$

with c being a dimensionless number of order one. The standard decoupling argument is based on the fact that such high dimension operators are suppressed by inverse powers of $|\Lambda_m|$ and therefore they do not affect the dynamics of the low energy theory.

Let us explore in more detail this fact and its relation to the one-loop computation of the effective potential described in section 2. There, we calculated the effect of supersymmetry breaking mass terms on the low energy effective potential of the pseudo-flat directions. In that computation we focused on the light fields, whose mass is of order μ (for simplicity, we set $h = 1$), and we neglected the modes with mass of order Λ_m . Can the effect of these modes, whose masses are also split by supersymmetry breaking, change our conclusion about the effective potential?

Our one-loop effective potential (2.14) is proportional to $|\mu^2|$, and is thus not real analytic in the parameter μ^2 appearing in the superpotential. This non-analyticity is because the modes that we integrated out become massless as $\mu \rightarrow 0$, so their contribution to the effective potential is singular there. On the other hand, corrections from heavier modes, whose masses are of order Λ_m , are necessarily real analytic in μ^2 . In particular, the leading correction from the microscopic theory to the mass of the pseudo-modulus must have coefficient $|\mu^2|^2/|\Lambda_m|^2 = |\mu^2 \epsilon^2| \ll |\mu^2|$. Such corrections are much smaller than our result from the low-energy macroscopic theory. One way to see that is to integrate out the

massive modes for $\mu = 0$ and summarize the effect in a correction to the Kähler potential as in (4.2). Then we can use this corrected Kähler potential with the tree level superpotential to find the effect on the pseudo-flat directions. These corrections are $\sim |\mu^2 \epsilon^2|$, and are negligible.

This fact is significant. Without knowing the details of the microscopic theory, we cannot determine these loop effects involving modes with mass $\sim \Lambda_m$. We cannot even determine the sign of the dimensionless coefficients like c in (4.2), and therefore we cannot determine whether they bend the pseudo-flat directions upward or downward. Fortunately, these effects which we cannot compute are smaller than the one loop effects in the low energy theory which we can compute. The latter have the effect of stabilizing our vacuum.

Of course, this discussion about the *irrelevance of irrelevant operators* which are suppressed by powers of Λ_m is obvious and trivial. However, in equation (3.4) we took into account a nonperturbative effect which leads to a superpotential which is suppressed by powers of Λ_m . We are immediately led to ask two questions. First, how come this non-renormalizable interaction is reliably computed even though it depends on Λ_m ? Second, given that we consider this interaction, why is it justified to neglect other terms as in (4.2) which are also suppressed by powers of Λ_m ?

Let us first address the first question. As in (3.1), Λ_m appears as a way to parameterize the infrared free gauge coupling g , at energy scales below $|\Lambda_m|$. This is conceptually different from the appearance of $|\Lambda_m|$ in (4.2), which has to do with effects from the microscopic theory, above the Landau pole scale. The superpotential (3.4) is generated by low energy effects and therefore it is correctly computed in the low energy effective theory. As a check, the resulting expectation value of Φ (3.5) is much smaller than Λ_m and therefore it is reliably calculated.

Let us now turn to the second question, of how we can neglect higher order corrections to the Kähler potential while keeping the superpotential (3.4). The leading contribution of such terms comes from corrections in the Kähler potential (4.2) of the schematic form $|\Phi|^4/|\Lambda_m|^2$. The leading effect of such corrections in the scalar potential are, schematically,

$$\Delta_K V_{\text{eff}} \sim \left| \frac{\mu^2 \Phi}{\Lambda_m} \right|^2 \sim |\mu^2 \epsilon^2| |\Phi|^2, \tag{4.3}$$

which for $|\epsilon| \ll 1$ are negligible corrections to the term (2.14) that we computed above. Higher order corrections to the Kähler potential are suppressed by even higher powers of $\frac{\Phi}{\Lambda_m}$, and are clearly negligible for $|\Phi| \ll |\Lambda_m|$. The correction (4.3) should be compared with the correction to the tree level potential from the superpotential (3.4), which is of the form

$$\Delta_W V_{\text{eff}} \sim \left| \frac{\mu^2 \Phi^{\frac{N_f - N}{N}}}{\Lambda_m^{\frac{N_f - 3N}{N}}} \right| \tag{4.4}$$

For $|\Phi| \gg |\Lambda_m \epsilon^{\frac{2N}{N_f - 3N}}|$ the correction due to the superpotential (4.4) is more important than the correction due to the Kähler potential (4.3). For smaller values of Φ both corrections are negligible. This answers our second question.

We conclude that the corrections due to the high energy theory and other modes at the scale Λ_m do not invalidate our conclusions. Our perturbative computations in section 2 and the nonperturbative computations in section 3 are completely under control and lead to the dominant contributions to the low energy dynamics.

5. Meta-stable vacua in SUSY QCD

In the preceding sections, we have gradually assembled the tools necessary for analyzing supersymmetry breaking in SQCD. Now let us put these tools to work. The model of interest is $SU(N_c)$ SQCD with scale Λ coupled to N_f quarks $Q_f, \tilde{Q}_g, f, g = 1, \dots, N_f$ (for a review, see e.g. [14]). We take for the tree-level superpotential

$$W = \text{Tr } m M, \quad \text{where} \quad M_{fg} = Q_f \cdot \tilde{Q}_g, \quad (5.1)$$

and m is a non-degenerate $N_f \times N_f$ mass matrix. This theory has N_c supersymmetric ground states with

$$\langle M \rangle = (\Lambda^{3N_c - N_f} \det m)^{\frac{1}{N_c}} \frac{1}{m} \quad (5.2)$$

All these supersymmetric ground states preserve baryon number and correspondingly the expectation values of all the baryonic operators vanish.

The mass matrix m can be diagonalized by a bi-unitary transformation. Its diagonal elements can be set to real positive numbers m_i . We will be interested in the case where the m_i are small and of the same order of magnitude. More precisely, we explore the parameter range

$$m_i \ll |\Lambda|; \quad \frac{m_i}{m_j} \sim 1 \quad (5.3)$$

We will consider the cases $N_f > N_c$. Then, in the limit $m_i \rightarrow 0$ with $\frac{m_i}{m_j} \sim 1$ the expectation values $\langle M \rangle$ in (5.2) approach the origin.

The region around the origin can be studied in more detail using the duality of [13] between our electric $SU(N_c)$ SQCD and a magnetic $SU(N_f - N_c)$ gauge theory with scale $\tilde{\Lambda}$, coupled to N_f^2 singlets M_{fg} and N_f magnetic quarks q_f and \tilde{q}_f in the fundamental and anti-fundamental representation of $SU(N_f - N_c)$. We will mostly limit ourselves to the free magnetic range $N_f < \frac{3}{2}N_c$ where the dual magnetic theory is IR free; higher values of N_f will be briefly discussed at the end of this section. In the free magnetic range, the metric on the moduli space is smooth around the origin. Therefore, the Kähler potential is regular there and can be expanded

$$K = \frac{1}{\beta} \text{Tr} (q^\dagger q + \tilde{q}^\dagger \tilde{q}) + \frac{1}{\alpha |\Lambda|^2} \text{Tr} M^\dagger M + \dots, \quad (5.4)$$

where the scale Λ appears because the field M is identified with the microscopic field in (5.1), of classical dimension two. The dimensionless coefficients α and β are positive real numbers of order one whose precise numerical values cannot be easily determined because they are not associated with the holomorphic information in the theory. Our quantitative answers will depend on α and β , but our qualitative conclusions will not.

The superpotential of the dual $SU(N_f - N_c)$ theory is [13]

$$W_{\text{dual}} = \frac{1}{\widehat{\Lambda}} \text{Tr } Mq\tilde{q} + \text{Tr } mM. \quad (5.5)$$

The dimensionful coefficient $\widehat{\Lambda}$ is related to the scales in the problem through [14]

$$\Lambda^{3N_c - N_f} \widetilde{\Lambda}^{3(N_f - N_c) - N_f} = (-1)^{N_f - N_c} \widehat{\Lambda}^{N_f} \quad (5.6)$$

The dimensionful parameters of the magnetic theory, $\widetilde{\Lambda}$ and $\widehat{\Lambda}$, are not uniquely determined by the information in the electric theory. This fact is related to the freedom to rescale the magnetic quarks q and \tilde{q} .⁴ Rescaling q and \tilde{q} has a number of effects. Obviously, it changes the value of β in the Kähler potential (5.4) and the value of $\widehat{\Lambda}$ in the superpotential (5.5). It also changes the relation between the electric baryons, $B = Q^{N_c}$ and $\widetilde{B} = \widetilde{Q}^{N_c}$, and their expressions in terms of the magnetic quarks, $q^{N_f - N_c}$ and $\tilde{q}^{N_f - N_c}$. Finally, $\widetilde{\Lambda}$ also changes (in such a way that the relation (5.6) is preserved), because this rescaling is anomalous under the magnetic gauge group $SU(N_f - N_c)$.

Using the freedom to rescale q and \tilde{q} , we can always set $\beta = 1$, but then we cannot compute both $\widehat{\Lambda}$ and $\widetilde{\Lambda}$ in terms of the electric variables. Alternatively, we can rescale the magnetic quarks to set $B = Q^{N_c} = q^{N_f - N_c}$ and $\widetilde{B} = \widetilde{Q}^{N_c} = \tilde{q}^{N_f - N_c}$. But then we cannot compute β (which is dimensionful). Below, we will find that the two choices are convenient in different settings.

Let us first consider the case of equal masses, $m_i = m_0$. As discussed in section 4, the higher order corrections to K in (5.4) are suppressed by powers of Λ and are not important near $M = q = \tilde{q} = 0$. Also, K is evaluated at $m_0 = 0$; higher order corrections are $\mathcal{O}(\frac{m_0^2}{\Lambda^2})$ and are negligible. Therefore, the theory based on the Kähler potential (5.4) and the superpotential (5.5) is the same as the model studied in section 3, with the parameters and fields related by the dictionary

$$\begin{aligned} \varphi = q, \quad \tilde{\varphi} = \tilde{q}, \quad \Phi = \frac{M}{\sqrt{\alpha\Lambda}}, \\ h = \frac{\sqrt{\alpha\Lambda}}{\widehat{\Lambda}}, \quad \mu^2 = -m_0\widehat{\Lambda}, \quad \Lambda_m = \widetilde{\Lambda}, \quad N = N_f - N_c \end{aligned} \quad (5.7)$$

Here we have chosen $\beta = 1$ and expressed our answers as functions of $\widetilde{\Lambda}$ and $\widehat{\Lambda}$. As a consistency check, notice that (5.2) becomes identical to the supersymmetric vacuum (3.5) discussed at the end of section 3, after applying the dictionary (5.7) and the identity (5.6).

An interesting special case is $N_f = N_c + 1$, where the magnetic gauge group is trivial. Here it is not natural to set $\beta = 1$. Instead, we scale q and \tilde{q} such that they are the same as the baryons $B = Q^{N_c}$ and $\widetilde{B} = \widetilde{Q}^{N_c}$ of the electric theory. Then, we should replace the kinetic term for the magnetic quarks in (5.4) with $\frac{1}{\beta|\Lambda|^{2N_c - 2}}(B^\dagger B + \widetilde{B}^\dagger \widetilde{B})$, where again, β is a positive dimensionless parameter which cannot be easily found. The superpotential of the theory is not that of (5.5), but instead, it is [12, 14]

$$W = \frac{1}{\Lambda^{2N_c - 1}}(\widetilde{B}^T M B - \det M) + \text{Tr } mM \quad (5.8)$$

⁴There is no such freedom to rescale M because it has a precise normalization in (5.1), and correspondingly, we identify m in the second term in (5.5) with the microscopic mass matrix m .

(Note the additional determinant term.) For $N_c > 2$ the determinant interaction is negligible near the origin and this theory is the same as the $N = 1$ version of the theory in section 2.

We can now essentially borrow all our results from sections 2 and 3. We thus conclude that, for N_f in the range $N_c + 1 \leq N_f < \frac{3}{2}N_c$, and for suitable tree-level quark masses, SUSY QCD has a meta-stable supersymmetry breaking ground state near the origin! In fact we have a compact moduli space of such meta-stable vacua, parameterized by the various massless Goldstone bosons.

It is surprising that we can establish that a meta-stable state exists in the strongly coupled region of the theory. Furthermore, we find the vacuum energy and the entire light spectrum around that meta-stable state up to two dimensionless numbers α and β (or alternatively, α and $\tilde{\Lambda}/\hat{\Lambda}$). Unlike other results in strongly coupled supersymmetric gauge theory, this result involves also non-supersymmetric and non-chiral information.

So far, we have derived this result for equal tree-level quark masses $m_i = m_0 \ll |\Lambda|$. But it is straightforward to generalize to unequal masses $m_i \ll |\Lambda|$. Consider first the approximation $|m_i - m_0| \ll m_0 \ll |\Lambda|$. Then, the effect of unequal masses is a small potential of order $m_i - m_0$ on the moduli space of our meta-stable vacua. Since this moduli space is compact, the theory with unequal masses also has a meta-stable vacuum.

More generally, for arbitrary $m_i \ll |\Lambda|$ we can still use our low energy effective field theory and conclude that a meta-stable state exists near the origin. For unequal masses $m_i \ll |\Lambda|$, the superpotential (2.4) of the macroscopic theory is replaced with $W_{\text{tree}} = h \text{Tr} \varphi \Phi \tilde{\varphi} - h \sum_{i=1}^{N_f} \mu_i^2 \Phi_i^i$, where $\mu_i^2 = -m_i \hat{\Lambda}$. We order the m_i so that $m_1 \geq m_2 \dots \geq m_{N_f} > 0$. The meta-stable vacuum is then given by

$$\Phi = 0, \quad \varphi = \tilde{\varphi}^T = \begin{pmatrix} \varphi_0 \\ 0 \end{pmatrix}, \quad \varphi_0 = \text{diag}(\mu_1, \mu_2, \dots, \mu_N). \quad (5.9)$$

In this vacuum, the non-vanishing F-terms are $F_{\Phi_i^i}$ for $i = N + 1, \dots, N_f$, and the vacuum energy is $V_0 = \sum_{i=N+1}^{N_f} |h\mu_i^2|$. For the vacuum (5.9) to be (meta) stable, it is crucial that the φ_0 expectation values in (5.9) are set by the N largest masses m_i . Replacing one of the φ_0 entries $\mu_{i \leq N}$ in (5.9) with a $\mu_{i > N}$ does not yield a (meta) stable vacuum — the tree level spectrum contains an unstable mode, sliding down to the vacuum (5.9).

What happens for m_i large compared with $|\Lambda|$? Clearly, our approximations can no longer be trusted. In particular, if all $m_i \gg |\Lambda|$ we have no reason to believe that such a meta-stable state exists. However, let us try to make one of the masses, m_{N_f} large while keeping the other masses small. For $m_{N_f} \gg |\Lambda|$ we can integrate out the heavy quark and reduce the problem to that of smaller number of flavors. As long as the number of light flavors \hat{N}_f satisfies $\hat{N}_f \geq N_c + 1$, our effective Lagrangian argument shows that such a meta-stable vacuum exists.

Let us try to go one step further and flow down from $N_f = N_c + 1 \rightarrow N_c$. We start with $N_c + 1$ light flavors with $m_{i=1 \dots N_c} \ll m_{N_c+1} \ll |\Lambda|$ and find a meta-stable state which up to symmetry transformations has $B_i = \tilde{B}_i = 0$, for all $i = 1 \dots N_c$, and $B_{N_c+1} = \tilde{B}_{N_c+1} \neq 0$. If we can trust this approximation as $m_{N_c+1} \gg |\Lambda|$, we find the following picture for the

$N_f = N_c$ problem. For $m = 0$ the low energy theory is characterized by the modified moduli space of vacua [12]

$$\det M - B\tilde{B} = \Lambda^{2N_c} \quad (5.10)$$

and the Kähler potential on that space is smooth. Consider the theory at the vicinity of the points related to

$$M = 0, \quad B = \tilde{B} = i\Lambda^{N_c} \quad (5.11)$$

by the action of the global baryon number symmetry. The Kähler potential around that point depends on the fields which are tangent to the constraint (5.10)

$$K = \frac{1}{\alpha|\Lambda|^2} \text{Tr } M^\dagger M + \frac{|\Lambda|^2}{\beta} b^\dagger b + \dots \quad (5.12)$$

where $B = i\Lambda^{N_c} e^b$, $\tilde{B} = i\Lambda^{N_c} e^{-b}$, and again α and β are dimensionless real and positive numbers which we cannot compute. Turning on the superpotential $m_0 \text{Tr } M$ leaves unlifted, to leading order, the pseudo-flat directions labelled by M and b . These pseudo-flat directions are lifted by the higher order terms in (5.12) which we cannot compute. (Note that unlike the case with more flavors, where the loops of massive but light fields give the dominant correction to the pseudo-flat directions, here there are no such light fields which can lead to a reliable conclusion.) Although we cannot prove it in this case, motivated by the flow from the problem with one more flavor, we suggest that the states (5.11) might also be meta-stable.

So far we have restricted attention to $N_f < \frac{3}{2}N_c$ where the magnetic degrees of freedom are IR free. What happens for larger values of N_f ? Clearly, for $N_f \geq 3N_c$ the electric theory is not strongly coupled in the IR and its dynamics is trivial. Therefore, our meta-stable states are not present. For $\frac{3}{2}N_c < N_f < 3N_c$ the theory flows to a nontrivial fixed point [13]. We can again use the magnetic description which flows to the same fixed point. However, the analysis above in the magnetic theory should be modified in this case. The duality is still valid only below Λ , but unlike the free magnetic case, here the magnetic theory is interacting in this range. A closely related fact is that, for nonzero M , the dynamically generated superpotential is [16, 14, 17]:

$$W_{\text{dyn}} = (N_c - N_f) \left(\frac{\det M}{\Lambda^{3N_c - N_f}} \right)^{\frac{1}{N_f - N_c}} \quad (5.13)$$

(One can check that this is the same as (3.4) after using (5.7) and (5.6).) For M near the origin, this scales like $M^{\frac{N_f}{N_f - N_c}}$ which is larger than M^3 and cannot be neglected in the analysis of the potential. Equivalently, for these values of N_f and N_c the expectation value of M (5.2) is too close to the origin to allow the existence of our meta-stable state. Finally, the case $N_f = \frac{3}{2}N_c$ is more subtle because the magnetic theory is IR free only because of its two loop beta function. Here the superpotential (5.13) scales like M^3 and again it cannot be neglected near the origin. It is interesting that in this case (5.13) is independent of Λ and in terms of the magnetic variables the superpotential (5.13) is independent of Λ_m .

To summarize, we have demonstrated in this section that $\text{SU}(N_c)$ SQCD with $N_c + 1 \leq N_f < \frac{3}{2}N_c$ massive flavors exhibits dynamical meta-stable supersymmetry breaking. In

addition, we have suggested that the same might be true for $N_f = N_c$. Our calculations are completely under control when the tree-level masses are in the regime $m_i \ll |\Lambda|$. The correction computed in section 2 due to integrating out light fields is of order $m_i/|\Lambda|$ and is the leading order correction to the effective potential.

If we take the masses m_i to all be equal, there is a vector-like $U(N_f) \cong SU(N_f) \times U(1)_B$ global symmetry. This symmetry is unbroken in the supersymmetric vacua (5.2), which is consistent with their mass gap. In the meta-stable, dynamical supersymmetry breaking vacua, the $U(N_f)$ global symmetry is spontaneously broken to $S(U(N_f - N_c) \times U(N_c))$ (plus there is an accidental $U(1)_R$ symmetry). The meta-stable dynamical supersymmetry breaking vacua is thus a compact moduli space of vacua,

$$\mathcal{M}_c \cong \frac{U(N_f)}{S(U(N_f - N_c) \times U(N_c))}. \tag{5.14}$$

Note that there is a bigger configuration space (5.14) of vacua with broken supersymmetry, versus the isolated supersymmetric vacua. Perhaps the larger configuration space will favor cosmology initially populating the vacua with broken supersymmetry.

Let us summarize the mass spectrum in the vacua with broken supersymmetry. There are many heavy states, associated with the microscopic theory, with masses of the order of Λ . The fields of the low-energy effective theory are those of the magnetic dual. Some of these fields get tree-level masses, of the order of $\sqrt{m\Lambda} \ll \Lambda$; this includes the magnetic gauge fields and gauginos, which are Higgsed. The pseudo-moduli have masses which are smaller, suppressed by a loop factor of the IR free Yukawa coupling of the magnetic dual. There are massless scalars: the Goldstone bosons of the vacuum manifold (5.14). There are also massless fermions (including the Goldstino): the N_c^2 fermionic partners of the pseudo-moduli Φ_0 , i.e. the fermions ψ_M in the null space of both $\langle q \rangle$ and $\langle \tilde{q} \rangle$.

We also note that the non-trivial topology of the vacuum manifold (5.14) means that there are topological solitons, whose lifetime is expected to be roughly the same as that of the meta-stable vacuum. In 4d, there are p -brane topological solitons if $\pi_{3-p}(\mathcal{M}_c)$ is non-trivial. In particular, the vacuum manifold (5.14) leads to solitonic strings.

6. $SO(N)$ and $Sp(N)$ generalizations

In this section, we give the generalizations of our models to $SO(N)$ and $Sp(N)$ groups. The $SO(N)$ theory (or more precisely, $Spin(N)$, so we can introduce sources in the spinor representation) exhibits a new phenomenon: the meta-stable, non-supersymmetric vacua are in the confining phase, whereas the supersymmetric vacua are in a different phase, the oblique confining phase. These different phases occur in this case because the dynamical matter is in an unfaithful representation of the center of the gauge group, leaving $\mathbb{Z}_2 \times \mathbb{Z}_2$ electric and magnetic order parameters which can not be screened. The order parameters determine whether Wilson and 't Hooft loops in the spinor representation of the $SO(N)$ group have area or perimeter law. We will argue that, in the meta-stable vacua with broken supersymmetry, the 't Hooft loop with magnetic \mathbb{Z}_2 charge has perimeter law, while that with oblique electric and magnetic \mathbb{Z}_2 charges has area law. In the supersymmetric vacua

the situation is reversed: the oblique charged loop has perimeter law, and the magnetic charged loop has area law.

6.1 The $SO(N)$ macroscopic theory

Consider a model with global symmetry and matter content

$$\begin{array}{ccccc}
 & SO(N) & SU(N_f) & U(1)' & U(1)_R \\
 \Phi & 1 & \square\square & -2 & 2 \\
 \varphi & \square & \square & 1 & 0
 \end{array} \tag{6.1}$$

The Kähler potential is taken to be canonical,

$$K = \text{Tr } \varphi^\dagger \varphi + \text{Tr } \Phi^\dagger \Phi \tag{6.2}$$

(Because Φ is a symmetric matrix, the Kähler potential has an extra factor of 2 for the off-diagonal components of Φ . This will be properly taken into account in the following analysis.) The superpotential is taken to be

$$W = h \text{Tr } \varphi^T \Phi \varphi - h\mu^2 \text{Tr } \Phi. \tag{6.3}$$

For $\mu \neq 0$, the $SU(N_f) \times U(1)'$ global symmetry is broken to $SO(N_f)$.

For $N_f > N$ and $\mu \neq 0$, supersymmetry is spontaneously broken as the rank condition again prevents F_Φ from all vanishing. Up to global symmetries, the potential is minimized by

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \end{pmatrix}, \quad \varphi = \begin{pmatrix} \varphi_0 \\ 0 \end{pmatrix}, \quad \text{with } \varphi_0^T \varphi_0 = \mu^2 \mathbb{1}_N \tag{6.4}$$

where Φ_0 is an arbitrary $(N_f - N) \times (N_f - N)$ symmetric matrix, and φ_0 is an $N \times N$ matrix subject to the condition in (6.4). All vacua on this space of classical pseudo-flat directions have degenerate vacuum energy density

$$V_{\min} = (N_f - N) |h^2 \mu^4|. \tag{6.5}$$

We can use the $SU(N)$ result of section 2 to show that (6.5) is indeed the absolute minimum of the potential. The classical potential of this $SO(N)$ theory satisfies $V_{SO(N)} \geq |h\varphi\varphi^T - h\mu^2|^2 \geq (N_f - N) |h^2 \mu^4|$, where for the first inequality we simply set $\Phi = 0$ and in the second we used the $SU(N)$ result, restricted to the smaller space where $\tilde{\varphi} = \varphi^T$.

We now show that perturbative quantum effects lift the above classical vacuum degeneracy, and that a local minimum of the one-loop effective potential is (up to symmetries)

$$\Phi_0 = 0, \quad \varphi_0 = \mu \mathbb{1}_N. \tag{6.6}$$

Of the classical vacua (6.4), this has maximal unbroken global symmetry, with $SO(N) \times SO(N_f) \times U(1)_R \rightarrow SO(N)_D \times SO(N_f - N) \times U(1)_R$. We will focus on the leading perturbative corrections to the effective potential, expanded around the vacuum (6.6).

Expanding around (6.6), we write the fields as

$$\Phi = \begin{pmatrix} \delta Y & \delta Z^T \\ \delta Z & \delta \widehat{\Phi} \end{pmatrix}, \quad \varphi = \begin{pmatrix} \mu + \delta\chi_A + \delta\chi_S \\ \delta\rho \end{pmatrix}. \quad (6.7)$$

where $\delta\chi_A$ and $\delta\chi_S$ denote the antisymmetric and symmetric part, respectively, of $\delta\chi_A + \delta\chi_S$. The Goldstone bosons of the broken global symmetry are $\text{Re} \left(\frac{\mu^*}{|\mu|} \delta\chi_A \right)$ and $\text{Re} \left(\frac{\mu^*}{|\mu|} \delta\rho \right)$. The former are in the adjoint of $\text{SO}(N) \times \text{SO}(N)_F / \text{SO}(N)_D \cong \text{SO}(N)$ (with $\text{SO}(N)_F \subset \text{SO}(N_f)$), and hence they are antisymmetric; the latter are in $\text{SO}(N_f) / \text{SO}(N)_F \times \text{SO}(N_f - N)$.

There are also the classically massless pseudo-moduli fields,

$$\delta \widehat{\Phi} \quad \text{and} \quad \delta \widehat{\chi} \equiv \text{Im} \left(\frac{\mu^*}{|\mu|} \delta\chi_A \right). \quad (6.8)$$

These are lifted at one-loop, with an effective potential that is constrained by the symmetries and dimensional analysis to have the form

$$V_{\text{eff}}^{(1)} = |h^4 \mu^2| \left(\frac{1}{2} a \text{Tr} \delta \widehat{\chi}^T \delta \widehat{\chi} + b \text{Tr} \delta \widehat{\Phi}^\dagger \delta \widehat{\Phi} \right) + \dots \quad (6.9)$$

for some numerical coefficients a and b . These coefficients are computed in appendix B; the calculation is very similar to the $\text{SU}(N)$ case. The result is

$$V_{\text{eff}}^{(1)} = \frac{|h^4 \mu^2| (\log 4 - 1)}{2\pi^2} \left((N_f - N) \text{Tr} \delta \widehat{\chi}^T \delta \widehat{\chi} + N \text{Tr} \delta \widehat{\Phi}^\dagger \delta \widehat{\Phi} \right) + \dots \quad (6.10)$$

The mass-squares of the pseudo-moduli are positive. The one-loop potential (6.10) stabilizes all (non-Goldstone-boson) pseudo-flat directions at the origin, with mass $\sim |h^2 \mu|$.

Now consider the effect of gauging $\text{SO}(N)$, taking it to be infrared free, $N_f > 3(N - 2)$. Then this theory becomes a macroscopic, low-energy effective theory, valid for energies below some cutoff scale Λ_m . The vacuum is still (6.6) since the D-terms vanish there. The gauge group is completely broken in the vacuum, so the $\text{SO}(N)$ vector bosons, together with the pseudo-moduli and Goldstone bosons derived from $\delta\chi_A$, acquire masses from the super-Higgs mechanism. Exactly as in the $\text{SU}(N)$ case, at leading order the tree-level $\text{SO}(N)$ vector supermultiplet masses are not split by the supersymmetry breaking. Thus, the one-loop potential for the remaining pseudo-modulus $\delta \widehat{\Phi}$ is the same as in (6.9).

Evidently, gauging $\text{SO}(N)$ does not significantly affect the non-supersymmetric vacuum (6.6). But just as for $\text{SU}(N)$, it does introduce a supersymmetric vacuum elsewhere in field space. Giving Φ general, non-zero expectation values in (6.3) gives the fields φ masses, and integrating them out leads to the low-energy effective superpotential

$$W_{\text{low}} = (N - 2) \left(h^{N_f} \Lambda_m^{3(N-2) - N_f} \det \Phi \right)^{1/(N-2)} - h\mu^2 \text{Tr} \Phi, \quad (6.11)$$

where the first term arises from gaugino condensation in the low-energy $\text{SO}(N)$ Yang-Mills theory, with scale related to Λ_m by matching at the scale $\langle h\Phi \rangle$ where φ are integrated out.

The first term in (6.11) leads to dynamical supersymmetry restoration, with $N_f - N + 2$ supersymmetric vacua at

$$\langle \Phi \rangle = \frac{\Lambda_m}{h} \epsilon^{2(N-2)/(N_f-N+2)} \mathbb{1}_{N_f}, \quad \text{where} \quad \epsilon \equiv \frac{\mu}{\Lambda_m}. \quad (6.12)$$

Again, we take $|\epsilon| \ll 1$ parametrically small to be able to reliably compute within the macroscopic effective theory. We will see that this also ensures that the meta-stable, non-supersymmetric vacuum (6.6) is long lived.

6.2 The ultraviolet theory: $\text{SO}(N_c)$ with $N_f < \frac{3}{2}(N_c - 2)$ massive flavors

The macroscopic theory of the previous subsection is the infrared free dual [18] of $\text{SO}(N_c)$ with $N_f < \frac{3}{2}(N_c - 2)$ massive flavors, which is asymptotically free. The dictionary relating the microscopic $\text{SO}(N_c)$ theory to the macroscopic $\text{SO}(N)$ theory (6.3) is much as in (5.7), except that here $N = N_f - N_c + 4$, and there are no $\tilde{\varphi}$ or \tilde{q} fields. The supersymmetric vacua (6.12) are those discussed in [18], and expected from the Witten index of $\text{SO}(N_c)$ with massive matter.

There are some special cases in the duality of [18]. For $N_f = N_c - 2$, the magnetic dual is $N = 2$, i.e. $\text{SO}(2)$; the infrared theory is then in the Coulomb phase. The theory (6.3) describes the N_f magnetic monopoles, ϕ^i , near $M_{ij} = Q_i \cdot Q_j = 0$. Actually, as mentioned in [18], the superpotential (6.3) in this case should be multiplied by a holomorphic function $f(t)$, with $t = \det M / \Lambda^{2(N_c-2)}$ and $f(0) = 1$. The leading order mass spectrum of the meta-stable, supersymmetry breaking vacuum involves only $f(0)$, and so it is completely independent of this function.

In the vacuum (6.4), the magnetic $\text{SO}(2)$ is Higgsed, and the unbroken electric $\text{SO}(2)$ is confined. For $N_f > 2$, these vacua break supersymmetry, and are meta-stable. The supersymmetric vacua of the electric theory with massive flavors comes from the massless dyon point of [18], at $\det M_{ij} = 16\Lambda^{2N_c-4}$; upon adding masses for the electric flavors, these dyons condense, and lead to the supersymmetric vacua (6.12). Condensing of the dyons leads to oblique confinement. We thus find that our meta-stable non-supersymmetric vacuum, and the supersymmetric vacua, are in different phases: confining, and oblique confining, respectively. Wilson and 't Hooft loops in the spinor representation can not be screened by the dynamical matter, so we have $\mathbb{Z}_2 \times \mathbb{Z}_2$ order parameters which can distinguish between the confining and oblique confining phases. The loop with area law in the non-supersymmetric vacuum will have perimeter law in the supersymmetric vacua, and vice versa. We expect that this is also true for $N_f > N_c - 2$, because we do not expect a phase transition if we give some flavors large masses, and flow down to $N_f = N_c - 2$.

For $N_f = N_c - 3$, there are two physically inequivalent phase branches [18]. The supersymmetric vacua of the theory with mass terms come from the branch with a dynamical superpotential $W_{\text{dyn}} \sim 1/\det M$. The other branch has the fields of (6.3) with $N = 1$, where $\text{SO}(1)$ means that there is one magnetic color index, but no corresponding gauge group. The superpotential (6.3) can in general be modified by multiplying it by a holomorphic function $f(t)$, with $t = \det M (M^{ij} q_i q_j) / \Lambda^{2N_c-3}$, with $f(0) = 1$ [18]. This branch

leads to our meta-stable non-supersymmetric vacua, with a spectrum that is independent of the function $f(t)$.

For $N_f = N_c - 4$, there are again two physically inequivalent branches, one with dynamical superpotential and one with $W_{\text{dyn}} = 0$ [18]. The branch with dynamical superpotential leads to the expected supersymmetric vacua upon adding mass terms for the flavors of the microscopic theory. On the other hand, the vacua with $W_{\text{dyn}} = 0$ break supersymmetry upon adding $W_{\text{tree}} = m \text{Tr} M \equiv -h\mu^2 \text{Tr} \Phi$ [18]. The leading Kähler potential near the origin is

$$K = \text{Tr} \Phi^\dagger \Phi + \frac{c}{|\Lambda_m|^2} \text{Tr}(\Phi^\dagger \Phi)^2 + \dots, \quad (6.13)$$

with c a number of order one. If c is negative (positive), the potential at the origin curves up (down). On the other hand, for large Φ the scalar potential must curve up, because there the Kähler potential must agree with the classical Kähler potential of the electric description, $K \sim \sqrt{M^\dagger M}$, with $M \sim \Phi$. So assuming that the Kähler potential is nondegenerate for all Φ , we conclude that there must exist a non-supersymmetric vacuum, *somewhere* on the $W_{\text{dyn}} = 0$ branch, regardless of the sign of c . This non-supersymmetric vacuum is stable on this branch of the theory; it can only decay via tunnelling to the $W_{\text{dyn}} \neq 0$ branch. The situation here should be compared with the analogous situation in $SU(N_c)$ SQCD with $N_f = N_c$. There we have only one branch, and therefore we cannot conclude definitively that there is a meta-stable vacuum. But the analogy with this $SO(N_c)$ example further motivates our suggestion above that such a meta-stable state exists.

6.3 $\text{Sp}(N)$ theories

The $\text{Sp}(N)$ theory is especially simple. It does not have the richness of the different phases of the $SO(N)$ theory and it does not have the baryons of the $SU(N)$ theory. Our conventions are such that $\text{Sp}(N)$ consists of all $A \in SU(2N)$ satisfying $A^T J_{2N} A = J_{2N}$, with $J_{2N} = \mathbb{1}_N \otimes (i\sigma_2)$. In particular, $\text{Sp}(1) \cong SU(2)$.

The macroscopic, low-energy theory has symmetries and matter content

	$\text{Sp}(N)$	$SU(2N_f)$	$U(1)'$	$U(1)_R$	
Φ	1	\square	-2	2	(6.14)
φ	\square	$\bar{\square}$	1	0	

canonical Kähler potential, and superpotential

$$W = h \text{Tr} \varphi^T \Phi \varphi J_{2N} - h\mu^2 \text{Tr} \Phi J_{2N_f}. \quad (6.15)$$

For $\mu \neq 0$, $SU(2N_f) \times U(1)'$ is broken to $\text{Sp}(N_f)$. We take $N_f > 3(N + 1)$, so the $\text{Sp}(N)$ gauge coupling is infrared free. Again, we first consider the theory for zero $\text{Sp}(N)$ gauge coupling.

The scalar potential has an absolute minimum, with energy density

$$V_{\text{min}} = 2(N_f - N)|h^2\mu^4|. \quad (6.16)$$

Indeed, we have $V_{\text{Sp}(N)} \geq |h(\varphi J_{2N} \varphi^T - \mu^2 J_{2N_f})|^2 \geq 2(N_f - N) |h^2 \mu^4|$, where for the first inequality we sent $\Phi = 0$, and in the second we used the result for $\text{SU}(2N)$, with $2N_f$ flavors, restricted to a smaller subspace where $\tilde{\varphi} = J_{2N} \varphi^T$. Up to unbroken global symmetries, the classical vacua with this minimum energy are

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \Phi_0 \end{pmatrix}, \quad \varphi = \begin{pmatrix} \varphi_0 \\ 0 \end{pmatrix}, \quad \text{with } \varphi_0 J_{2N} \varphi_0^T = \mu^2 J_{2N} \quad (6.17)$$

where Φ_0 is an arbitrary $2(N_f - N) \times 2(N_f - N)$ antisymmetric matrix, and φ_0 is a $2N \times 2N$ matrix.

The one-loop effective potential lifts this classical vacuum degeneracy, and the local minimum is at the point of maximal unbroken global symmetry:

$$\Phi_0 = 0, \quad \varphi_0 = \mu \mathbb{1}_{2N} \quad (6.18)$$

which leaves unbroken a $\text{Sp}(N)_D \times \text{Sp}(N_f - N) \times \text{U}(1)_R$ global symmetry. Let us decompose the fluctuations around this point as

$$\Phi = \begin{pmatrix} \delta Y & \delta Z^T \\ -\delta Z & \delta \hat{\Phi} \end{pmatrix}, \quad \varphi = \begin{pmatrix} \mu + J_{2N}(\delta \chi_A + \delta \chi_S) \\ \delta \rho \end{pmatrix} \quad (6.19)$$

where again by $\delta \chi_A$ and $\delta \chi_S$ we mean the antisymmetric and symmetric part, respectively, of $\delta \chi_A + \delta \chi_S$. The Goldstone bosons of the broken global symmetry are

$$\frac{\mu^*}{|\mu|} \delta \chi_S - \frac{\mu}{|\mu|} J_{2N} \delta \chi_S^* J_{2N} \quad \text{and} \quad \frac{\mu^*}{|\mu|} \delta \rho + \frac{\mu}{|\mu|} J_{2(N_f - N)} \delta \rho^* J_{2N} \quad (6.20)$$

The former are in the adjoint of $\text{Sp}(N) \times \text{Sp}(N)_F / \text{Sp}(N)_D \cong \text{Sp}(N)$, and hence symmetric; the latter are in $\text{Sp}(N_f) / \text{Sp}(N)_F \times \text{Sp}(N_f - N)$. There are also classically massless pseudo-moduli fields

$$\delta \hat{\Phi} \quad \text{and} \quad \delta \hat{\chi} \equiv \frac{\mu^*}{|\mu|} \delta \chi_S + \frac{\mu}{|\mu|} J_{2N} \delta \chi_S^* J_{2N} \quad (6.21)$$

Once again, the global symmetries and dimensional analysis constrain the one-loop effective potential to have the form

$$V_{\text{eff}}^{(1)} = |h^4 \mu^2| \left(\frac{1}{2} a \text{Tr} (J_{2N} \delta \hat{\chi})^2 + b \text{Tr} \delta \hat{\Phi}^\dagger \delta \hat{\Phi} \right) + \dots \quad (6.22)$$

The coefficients a and b are computed in appendix B, and the result is

$$V_{\text{eff}}^{(1)} = \frac{|h^4 \mu^2| (\log 4 - 1)}{\pi^2} \left(\frac{1}{4} (N_f - N) \text{Tr} (J_{2N} \delta \hat{\chi})^2 + N \text{Tr} \delta \hat{\Phi}^\dagger \delta \hat{\Phi} \right). \quad (6.23)$$

The pseudo-flat directions are thus stabilized, with non-tachyonic masses $\sim |h^2 \mu|$.

As in the $\text{SU}(N)$ and the $\text{SO}(N)$ examples, gauging $\text{Sp}(N)$ does not affect the one-loop potential (6.23), because the classical masses of the (completely Higgsed) $\text{Sp}(N)$ vector multiplet are supersymmetric. And, as above, gauging $\text{Sp}(N)$ leads to supersymmetric vacua, by dynamical supersymmetry restoration, elsewhere in field space. The supersymmetry-breaking vacua (6.18) are thus meta-stable.

The theory (6.15) is the dual of a microscopic theory given by $\text{Sp}(N_c)$ gauge theory, with $2N_f$ fundamental flavors Q [19]. The dictionary is much as in (5.7), except that $N = N_f - N_c - 2$. For $N_f < \frac{3}{2}(N_c + 1)$, the macroscopic $\text{Sp}(N)$ theory (6.15) is infrared free. Our analysis of the macroscopic theory shows that the microscopic $\text{Sp}(N_c)$ theory, with small masses for the fundamental flavors, has the meta-stable, supersymmetry breaking vacua, given by (6.17) and (6.18).

For $N_f = N_c + 2$, we have $N = 0$, so the dual theory does not include gauge fields. The fields of the low-energy theory are just M , with a superpotential [19],

$$W = -\frac{\text{Pf}M}{\Lambda^{2N_c+1}} + \text{Tr} m M. \tag{6.24}$$

In many respects this case is similar to the $\text{SU}(N_c = N_f - 1)$ theories. However, unlike these theories the superpotential (6.24) does not include cubic terms (for $\text{Sp}(N_c > 1)$), and therefore only $\text{Tr} m M$ is important near the origin. Then, depending on the Kähler potential, this term could lead to a supersymmetry breaking meta-stable state near the origin. In this respect this situation is similar to the $\text{SU}(N_c = N_f)$ theories.

Finally, we can analyze the case where one mass eigenvalue is much larger than the others, flowing to $N_f = N_c + 1$, where there is a quantum modified moduli space constraint [19], analogous to that of $\text{SU}(N_c = N_f)$ SQCD. The analysis of the theory with mass terms in analogous to the discussion following (5.10), with some components of M here playing the role of the baryon expectation values in (5.11) ($\text{Sp}(N)$ does not have baryons). Again, we suggest here that for $N_f = N_c + 1$ and small tree-level masses, $\text{Sp}(N_c > 1)$ SQCD has meta-stable supersymmetry-breaking vacua near the origin of field space.

7. Estimating the lifetime of the meta-stable vacua

In this section, we will show that our meta-stable, non-supersymmetric vacua can be made parametrically long lived, by taking the parameter $\epsilon \equiv \mu/\Lambda_m \sim \sqrt{m/\Lambda}$ to be sufficiently small. We ignore quantum gravity effects,⁵ and consider only the semi-classical field theory decay as in [21]. The semi-classical decay probability, which sets the decay rate, is given by $\exp(-S)$, where S is the “bounce” action (the difference between the Euclidean action of the tunneling configuration and that of remaining in the meta-stable vacuum), times an irrelevant one-loop prefactor. We will argue that S is parametrically large as $\epsilon \rightarrow 0$, making the lifetime arbitrarily long.

In order to give a qualitative estimate of the bounce action S , we need to give a qualitative picture of the potential for the scalar fields, Φ and φ and $\tilde{\varphi}$. Recall that our meta-stable non-supersymmetric vacuum is (we discuss the $\text{SU}(N)$ case; the discussion for $\text{SO}(N)$ and $\text{Sp}(N)$ is completely analogous):

$$\Phi = 0, \quad \varphi = \tilde{\varphi}^T = \begin{pmatrix} \mu \mathbb{1}_N \\ 0 \end{pmatrix}, \quad V_+ = (N_f - N) |h^2 \mu^4|. \tag{7.1}$$

⁵If we add a constant superpotential, so that the meta-stable vacuum has our observed vacuum energy, then the supersymmetric vacua are anti-deSitter. This can lead to a suppressed quantum gravity tunneling rate.

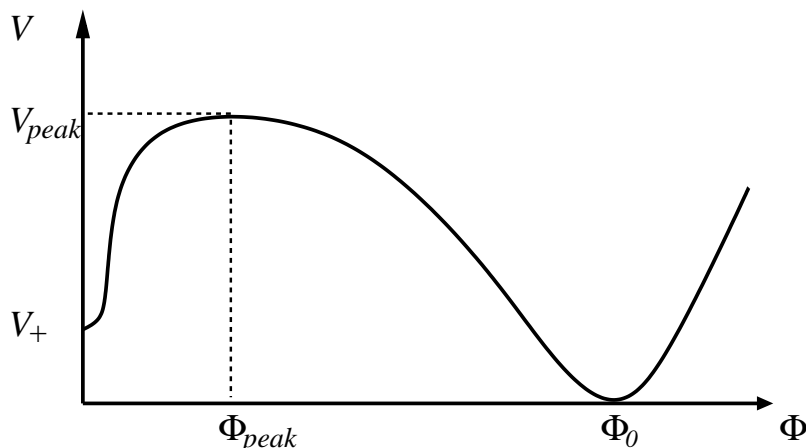


Figure 1: The potential along the bounce trajectory. The peak is at $\Phi_{\text{peak}} \sim \mu$ and the supersymmetric minimum with vanishing potential is at large field $\Phi_0 \sim \mu/\epsilon^{(N_f-3N)/(N_f-N)} \gg \mu$. The values of the potential at the local minimum V_+ and at the peak V_{peak} are of order μ^4 .

The supersymmetric vacuum (3.5) on the other hand has

$$\Phi = \frac{\mu}{h} \frac{1}{\epsilon^{(N_f-3N)/(N_f-N)}} \mathbb{1}_{N_f}, \quad \varphi = \tilde{\varphi} = 0, \quad V_0 = 0. \quad (7.2)$$

Because we take $N_f > 3N$, which is the condition for the macroscopic theory to be infrared free, the supersymmetric minimum (7.2) is parametrically far away from the meta-stable non-supersymmetric vacuum (7.1) as $\epsilon \rightarrow 0$. As we shall see, this large distance $\Delta\Phi$ in field space guarantees a parametrically large bounce action S .

The bounce action is expected to come from the path in field space with the least potential barrier between the vacua (7.1) and (7.2). Computing the classical potential from (2.4), we find terms $V_{\text{cl}} \supset |h\varphi\Phi|^2 + |h\Phi\tilde{\varphi}|^2$, which provide a large potential energy cost to having both Φ and φ or $\tilde{\varphi}$ being non-zero. The most efficient path is thus to climb quickly from (7.1) up to a point near the local peak

$$\Phi = 0, \quad \varphi = \tilde{\varphi} = 0, \quad V_{\text{peak}} = N_f |h^2 \mu^4|. \quad (7.3)$$

From there, we can take the path of increasing Φ , toward the minimum (7.2), keeping $\varphi = \tilde{\varphi} = 0$; the potential along this path is extremely flat, as $\epsilon \rightarrow 0$, sloping only very gently⁶ toward the minimum (7.2). A schematic picture of the potential is shown in figure 1.

The thin wall approximation [21] is not appropriate for computing the bounce action of such a potential. The needed calculation of the bounce action can be modelled by a triangle potential barrier. Then, using the results of [22] we find

$$S \sim \frac{(\Delta\Phi)^4}{V_+} \sim \frac{1}{|\epsilon|^{4(N_f-3N)/(N_f-N)}} \gg 1. \quad (7.4)$$

Taking $\epsilon \rightarrow 0$, we can make the minimal bounce action arbitrarily large, and thus make the meta-stable vacuum arbitrarily long lived.

⁶This gentle slope could be also useful for inflation or quintessence.

It is amusing to consider the very different magnification scale of the potential in the microscopic description of the theory and in the macroscopic description. The relation (7.4) applies in both descriptions. In the macroscopic description, we have $\epsilon = \mu/\Lambda_m$, with μ held fixed and the cutoff scale $\Lambda_m \rightarrow \infty$. Here the large action (7.4) is intuitive: the vacua (7.1) and (7.2) appear widely separated in field space. On the other hand, in the microscopic description, we have $\epsilon \sim \sqrt{m/\Lambda}$, and we hold Λ fixed and take m to zero. Here we are looking at the potential with a very different magnification scale, and the parametrically large action is less intuitive: the vacua (7.1) and (7.2) appear as tiny features, two close vacua separated by a tiny barrier. Nevertheless, the bounce action only depends on the ratio ϵ , not the overall scale μ , so the expression (7.4) remains valid. The decay rate of the meta-stable vacuum can be made exponentially parametrically small, by taking ϵ sufficiently small, whether we are in the macroscopic scaling where the features of the potential appear large, or in the microscopic scaling where they appear small.

8. Preliminary thoughts about model building

This work was motivated by attempts to find new models of supersymmetry breaking and new mechanisms to communicate supersymmetry breaking to the Standard Model. We hope that the theories studied in this paper are a modest step towards building a simpler and more elegant model of dynamical supersymmetry breaking. Of course, many challenges lie ahead, and we have not succeeded in overcoming these challenges. But we would like to share some of our preliminary ideas about them.

1. *Naturalness.* The small parameter which controls our approximations is $\epsilon \sim \sqrt{m/\Lambda}$ and the vacuum energy⁷ is proportional to $|m^2\Lambda^2|$. Since it is proportional to a power of Λ , it is nonperturbative. However, since it is also proportional to the tree level parameter m , our model does not satisfy the purist's requirement that all low energy scales are dynamically generated. Therefore, we would like to find other theories, using the same ideas as in our models, where the role of the parameter m is played by some marginal or irrelevant coupling constants. For example, we can imagine that the microscopic theory has such an operator suppressed by a power of the Planck scale (or some other high energy scale), $\frac{\lambda}{M_p^\Delta} \mathcal{O}$ with $\lambda \sim 1$. If this operator acquires a dynamical F-term $F_{\mathcal{O}} \sim \Lambda^{2+\Delta}$, then the vacuum energy is of order $\frac{\lambda^2 \Lambda^{4+2\Delta}}{M_p^{2\Delta}}$. This way supersymmetry is broken at a naturally small scale.⁸
2. *Direct mediation.* A longstanding goal of SUSY phenomenology, first discussed in [3] and later analyzed by various authors (see e.g. [8] and references therein), is to find a

⁷Of course, the actual vacuum energy density includes a negative supergravity contribution from the value of the superpotential in the minimum.

⁸Other models which are worth exploring are based on similar dualities, e.g. those of [23–25]–[26]. These theories contain many operators \mathcal{O} with large dimension Δ_0 at weak gauge coupling, but dimension $\Delta = 1$ in the infrared, where they are free. Adding them to the superpotential could lead to meta-stable, non-supersymmetric vacua, by an argument completely analogous to our rank condition in the dual theory. A more detailed analysis is needed to determine whether the interacting sector of the infrared theory changes our conclusions about the meta-stable non-supersymmetric vacuum.

simple model of *direct mediation* of supersymmetry breaking in which the standard model gauge group couples directly to the supersymmetry breaking sector. The basic idea of direct mediation is that the supersymmetry breaking sector has a large global symmetry G and a subgroup of it $H \subset G$ is gauged and is identified with (part of) the standard model gauge group. One of the hallmarks of our theories is that they have large global symmetries G which could be used this way.

Consider, for example, gauging the $SU(N_f)$ symmetry of our SUSY QCD example in section 3. Then, the gauge group below the scale Λ is $SU(N_f - N_c) \times SU(N_f)$ where the $SU(N_f - N_c)$ gauge theory is dual to the microscopic $SU(N_c)$ theory. In our meta-stable vacuum this symmetry is broken $SU(N_f - N_c) \times SU(N_f) \rightarrow SU(N_f - N_c) \times SU(N_c)$ where the first factor is embedded diagonally in $SU(N_f - N_c) \times SU(N_f)$, and the second factor is a subgroup of $SU(N_f)$. It is interesting that some of the low energy gauge fields are partially electric and partially magnetic. In the context of direct mediation of supersymmetry breaking we can think of this low energy gauge group (or a subgroup of it) as included in the standard model. Clearly, depending on the details of such a construction, we might need to abandon simple unification.

An obstacle for direct mediation is that, if we identify a subgroup of the standard model, e.g. the color $SU(3)_c$ symmetry with a subgroup H of the flavor symmetry G of the supersymmetry breaking sector, the colors of that sector lead to additional $SU(3)$ flavors. If there are too many such flavors, $SU(3)_c$ can have a Landau pole at a dangerously low scale. We do not have a solution to this problem. But we would like to suggest that the theory viewed at low energies as $SU(3)$ could be related in a complicated way to a more microscopic gauge symmetry. (In the particular example of the previous paragraph, however, this does not actually help.)

3. *R-symmetry problem.* Models of dynamical supersymmetry breaking with no supersymmetric vacua must either have a non-generic superpotential, or must have global $U(1)_R$ symmetry [3, 10]. However, in order to have nonzero Majorana gluino masses this R-symmetry should be broken, and to avoid a massless Goldstone boson this R-symmetry should be explicitly broken. This explicit breaking could restore supersymmetry. The authors of [27] pointed out that this problem can be solved using gravitational interactions. In our theories there is no exact R-symmetry and hence there exist supersymmetric vacua. But the existence of an accidental R-symmetry near the origin leads to a supersymmetry breaking meta-stable state. The small effect of the explicit $U(1)_R$ breaking in this meta-stable state might be strong enough to avoid the R-symmetry problem.⁹

It is interesting to compare our models with the discussion of “R-color” in [3], which is a non-Abelian gauge theory that was introduced in order to explicitly break the $U(1)_R$ symmetry. Our models in section 3 fit that pattern. The theory of section 2 has an

⁹M. Dine has pointed out to us that as it stands our theory has a discrete R-symmetry which prevents gluino masses. However, such a symmetry can be explicitly broken, e.g. by adding nonrenormalizable baryon operators in the microscopic superpotential.

R-symmetry and it breaks supersymmetry with a stable minimum. The $SU(N)$ gauge interactions added in section 3 explicitly break the R-symmetry, and they also introduce a supersymmetric state far in field space. Such supersymmetry restoration is a common phenomenon with R-color and was often considered a problem. However, the microscopic theory of section 5 gives another perspective on the issue. Here the R-symmetry is broken in the $SU(N_c)$ microscopic theory. In the meta-stable state, the $SU(N_c)$ gauge interactions dynamically break supersymmetry, and they also break the R-symmetry. The role of R-color is played by their magnetic dual, the $SU(N)$ gauge fields.

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A. F-term supersymmetry breaking

A.1 Generalities

Spontaneous supersymmetry breaking requires an exactly massless Goldstino fermion ψ_X . In simple models it originates from a chiral superfield X . The scalar component X can get a mass from either non-canonical Kähler potential terms, or more generally from corrections to the X propagator from loops of massive fields. Consider, a theory of a single chiral superfield X , with linear superpotential with coefficient f (with units of mass^2),

$$W = fX, \tag{A.1}$$

and effective Kähler potential $K(X, X^\dagger)$. Supersymmetry is spontaneously broken by the expectation value of the F-component of X . The potential, $V = K_{XX^\dagger}^{-1} |f|^2$, is non-vanishing as long as the Kähler metric is non-singular. The fermion ψ_X is the exactly massless Goldstino. If $K = K_{\text{can}} = XX^\dagger$, then the scalar component of X is also massless; the potential is $V = |f|^2$, independent of $\langle X \rangle$, so there are classical vacua for any $\langle X \rangle$. This vacuum degeneracy is lifted by any non-trivial Kähler potential. For example, if near the origin $K = XX^\dagger - \frac{c}{|\Lambda|^2} (XX^\dagger)^2 + \dots$, then there is a stable supersymmetric vacuum at the origin if $c > 0$. In this vacuum, the scalar component of X gets mass $m_X^2 \approx 4c|f|^2/|\Lambda|^2$. If $c < 0$, the origin is not the minimum of the potential.

The macroscopic, low-energy effective field theory must be under control to determine whether or not supersymmetry is broken. For example, $SU(2)$ with an $I = 3/2$ matter

field has an effective low energy superpotential (A.1). If the low energy theory is a free theory of a composite field X , as is suggested by non-trivial 't Hooft anomaly matching, supersymmetry is spontaneously broken. If instead the low energy theory is an interacting conformal theory, supersymmetry is unbroken [28].

In the example (A.1), a singularity in the Kähler metric signals the need to include additional light degrees of freedom. Suppose that an additional field q becomes massless at a particular value of X , which we can take to be $X = 0$, so

$$W = hXqq + fX. \quad (\text{A.2})$$

For $f = 0$, there is a moduli space of supersymmetric vacua, labelled by $\langle X \rangle$, and q can be integrated out away from the origin. Turning on f lifts this moduli space, but the theory no longer breaks supersymmetry, as there is a supersymmetric vacuum at $X = 0$, $q = \sqrt{-f/h}$. To determine whether or not supersymmetry is broken requires that the macroscopic low-energy theory be correctly identified.

In this paper, we will be interested in the one-loop effective potential for pseudo-moduli (such as X), which comes from computing the one-loop correction (1.4) to the vacuum energy. In (1.4), \mathcal{M}^2 stands for the classical mass-squareds of the various fields of the low-energy effective theory. For completeness, we recall the standard expressions for these masses. For a general theory with n chiral superfields, Q^a , with canonical classical Kähler potential, $K_{\text{cal}} = Q_a^\dagger Q^a$, and superpotential $W(Q_a)$:

$$m_0^2 = \begin{pmatrix} W^{\dagger ac} W_{cb} & W^{\dagger abc} W_c \\ W_{abc} W^{\dagger c} & W_{ac} W^{\dagger cb} \end{pmatrix}, \quad m_{1/2}^2 = \begin{pmatrix} W^{\dagger ac} W_{cb} & 0 \\ 0 & W_{ac} W^{\dagger cb} \end{pmatrix}, \quad (\text{A.3})$$

with $W_c \equiv \partial W / \partial Q^c$, etc., and m_0^2 and $m_{1/2}^2$ are $2n \times 2n$ matrices.

A.2 The basic O’Raifeartaigh model

The basic model has three chiral superfields, X , ϕ_1 , and ϕ_2 , with classical Kähler potential $K_{\text{cl}} = X^\dagger X + \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2$, and superpotential

$$W = \frac{1}{2} h X \phi_1^2 + h m \phi_1 \phi_2 - h \mu^2 X. \quad (\text{A.4})$$

We denote the coefficient f of the linear term as $f = -h\mu^2$, with μ having dimensions of mass, to make the mass dimension explicit, and to simplify expressions. This theory has a $U(1)_R$ symmetry, with $R(X) = 2$, $R(\phi_1) = 0$, $R(\phi_2) = 2$. The tree-level potential for the scalars is, $V_{\text{tree}} = |F_X|^2 + |F_{\phi_1}|^2 + |F_{\phi_2}|^2$, with

$$F_X = h \left(\frac{1}{2} \phi_1^2 - \mu^2 \right), \quad F_{\phi_1} = h (X \phi_1 + m \phi_2), \quad F_{\phi_2} = h m \phi_1. \quad (\text{A.5})$$

Supersymmetry is broken because F_X and F_{ϕ_2} cannot both vanish. The X and ϕ_2 equations of motion require that $F_{\phi_1} = 0$, which fixes $\langle \phi_2 \rangle = -\langle X \phi_1 / m \rangle$. The minimum of the potential is a moduli space of degenerate, non-supersymmetric vacua, with $\langle X \rangle$ arbitrary. The minimum of the potential depends on the parameter

$$y \equiv \left| \frac{\mu^2}{m^2} \right| \quad (\text{A.6})$$

For $y \leq 1$, the potential is minimized, with value $V = |h^2 \mu^4|$, at $\phi_1 = \phi_2 = 0$ and arbitrary X . There is a second order phase transition at $y = 1$, where this minimum splits to two minima and a saddle point. For $y \geq 1$ the potential has minima with $V = |h^2 \mu^4| \left(\frac{2y-1}{y^2} \right)$ at $\phi_1 = \pm i \sqrt{2\mu^2(1-1/y)}$, $\phi_2 = -X\phi_1/m$ with arbitrary X . Let us focus on the $y \leq 1$ phase.

The fermion ψ_X is the exactly massless Goldstino. The scalar component of X is a classically pseudo-modulus. The classical mass spectrum of the ϕ_1 and ϕ_2 field can be computed from (A.3). For the fermions, the eigenvalues are

$$m_{1/2}^2 = \frac{1}{4} |h|^2 (|X| \pm \sqrt{|X|^2 + 4|m|^2})^2, \tag{A.7}$$

and for the real scalars the mass eigenvalues are

$$m_0^2 = |h|^2 \left(|m|^2 + \frac{1}{2} \eta |\mu^2| + \frac{1}{2} |X|^2 \pm \frac{1}{2} \sqrt{|\mu^4| + 2\eta |\mu^2| |X|^2 + 4|m|^2 |X|^2 + |X|^4} \right), \tag{A.8}$$

where $\eta = \pm 1$. At $y = 1$, where the second order phase transition occurs, one of the eigenvalues (A.8) vanishes for all X : the otherwise massive fields from ϕ_1 and ϕ_2 yield an additional, classically massless, real scalar.

The classical flat direction of the classical pseudo-modulus X is lifted by a quantum effective potential, $V_{\text{eff}}(X)$. The one-loop effective potential can be computed from the expression (1.4) for the one-loop vacuum energy, using the classical masses (A.7) and (A.8). The pseudo-modulus X is here treated as a background. It is found that the resulting effective potential is minimized at $\langle X \rangle = 0$, so we'll simplify the expressions by just expanding around this minimum: $V_{\text{eff}} = V_0 + m_X^2 |X|^2 + \dots$. The one loop corrected vacuum energy is

$$V_0 = |h^2 \mu^4| \left[1 + \frac{|h^2|}{64\pi^2} \left(y^{-2}(1+y)^2 \log(1+y) + y^{-2}(1-y)^2 \log(1-y) + 2 \log \frac{|hm|^2}{\Lambda^2} \right) \right]. \tag{A.9}$$

The dependence on the cutoff Λ can be absorbed into the running h . The one-loop quantum mass of the classical pseudo-modulus X is given by

$$m_X^2 = + \frac{|h^4 \mu^2|}{32\pi^2} y^{-1} (-2 + y^{-1}(1+y)^2 \log(1+y) - y^{-1}(1-y)^2 \log(1-y)). \tag{A.10}$$

The mass (A.10) indeed satisfies $m_X^2 > 0$, consistent with the minimum of the one-loop potential (1.4) being at the origin. For small supersymmetry breaking, $y \rightarrow 0$, we have

$$m_X^2 \rightarrow \frac{|h^4 \mu^4|}{48\pi^2 |m|^2}, \quad \text{for} \quad |\mu^2| \ll |m|^2. \tag{A.11}$$

In the limit, $y \rightarrow 1$, where the supersymmetry breaking is large, we have

$$m_X^2 = \frac{|h^4 \mu^2|}{16\pi^2} (\log 4 - 1) \quad \text{for} \quad |\mu^2| = |m|^2. \tag{A.12}$$

Because the potential is minimized at $\langle X \rangle = 0$, the vacuum has broken supersymmetry but unbroken $U(1)_R$ symmetry. If the superpotential contains all terms allowed by

symmetries, then having a $U(1)_R$ symmetry is a necessary condition for supersymmetry breaking, and having $U(1)_R$ spontaneously broken is a sufficient condition for supersymmetry breaking [10]. Here we find that the correct quantum vacuum is actually that where $U(1)_R$ symmetry is not spontaneously broken, but supersymmetry is nevertheless broken.

When the supersymmetry breaking mass splittings are small, the effective potential can alternatively be computed in the supersymmetric low-energy effective theory where we integrate out the massive fields ϕ_1 and ϕ_2 . The effective superpotential of the low-energy theory is $W_{\text{low}} = -h\mu^2 X$, and the effective Kähler potential, $K_{\text{eff}}(X, X^\dagger)$, gets a one-loop correction from integrating out the massive fields. This gives the effective potential

$$V^{(1)} = (K_{\text{eff}}|_{X, X^\dagger})^{-1} |h^2 \mu^4|. \tag{A.13}$$

This way of computing the effective potential is valid only when the supersymmetry breaking is small, because the true effective potential generally gets significant additional contributions from terms that involve higher super-derivatives in superspace. The effective potential (1.4) gives the full answer, whether or not the supersymmetry breaking is small. In particular, (A.13) only reproduces the effective potential (1.4) to leading order in the $y \rightarrow 0$ limit. For example, (A.13) reproduces the mass (A.11) of the small supersymmetry breaking limit, but not the mass (A.12) of the large supersymmetry breaking limit. In appendix A.5 we prove, for generalized theories of tree-level supersymmetry breaking, that the potential (A.13), computed from the effective Kähler potential, always agrees with the order $|f|^2$ truncation of the correct effective potential, computed via (1.4).

A.3 Some closely related examples

Consider a theory of $2n + 1$ chiral superfields, X , and A_i, B_i , with $i = 1 \dots n$, Kähler potential $K = X^\dagger X + \sum_i A_i^\dagger A_i + B_i^\dagger B_i$, and superpotential

$$W = fX + \sum_i \left(\frac{1}{2} h_i X A_i^2 + h_i m_i A_i B_i \right). \tag{A.14}$$

This is not quite the same as n decoupled copies of the O’Raifeartaigh model (A.4), because the same chiral superfield X participates in each of them. Taking all $y_i \equiv |f/h_i m_i^2| \leq 1$, the classical vacuum is at $\langle A_i \rangle = \langle B_i \rangle = 0$, with $\langle X \rangle$ arbitrary and $V_{\text{tree}} = |f|^2$. The fermion ψ_X is exactly massless, and the scalar component of X gets mass starting at one-loop. The one-loop effective potential is computed from the vacuum energy (1.4), using the classical mass spectrum computed as a function of $\langle X \rangle$. The classical masses of A_i and B_i come from expanding V_{tree} to quadratic order in the A_i and B_i fields (the general formula is given in (A.3)). For example, for the scalars, we have

$$V_{\text{tree}} \supset \sum_i \left(\text{Re}(f^* h_i A_i^2) + |h_i m_i A_i|^2 + \left| h_i m_i B_i + \frac{1}{2} h_i A_i X \right|^2 \right). \tag{A.15}$$

These masses are the same as in the original O’Raifeartaigh model (A.4), for each flavor i ; the fermion masses are likewise simply a sum of those of the model (A.4), for each flavor i .

For each flavor i , the mass-squared eigenvalues are thus as in (A.7) and (A.8), and the one-loop effective potential (1.4) is simply a sum over i of that of the original model (A.4); so the minimum of the effective potential is again at $\langle X \rangle = 0$. In particular, the one-loop quantum mass of X is given (with $y_i \equiv |f/h_i m_i^2|$) by

$$m_X^2 = \sum_{i=1}^n \frac{|h_i^3 f|}{32\pi^2} y_i^{-1} (-2 + y_i^{-1}(1 + y_i)^2 \log(1 + y_i) - y_i^{-1}(1 - y_i)^2 \log(1 - y_i)). \quad (\text{A.16})$$

As another example, consider a theory of $2N$ chiral superfields S_i and V_i , $i = 1 \dots N$, with $K = S^{i\dagger} S_i + V^{i\dagger} V_i$ and superpotential

$$W = m S_i V_i, \quad \text{subject to} \quad V_i V_i = \Lambda^2. \quad (\text{A.17})$$

There is an $\text{SO}(N) \times \text{U}(1)_R$ global symmetry, with $R(S_i) = 2$ and $R(V_i) = 0$. It is impossible for $F_{S_i} = m V_i$ to all vanish, because of the constraint $V_i V_i = \Lambda^2$, so supersymmetry is broken. The constraint also spontaneously breaks the $\text{SO}(N)$ flavor symmetry to $\text{SO}(N-1)$, so there are $N-1$ massless Goldstone bosons. Solving the constraint equation, we can take $\vec{V} \equiv (\sqrt{\Lambda^2 - \vec{\phi}_1 \cdot \vec{\phi}_1}, \vec{\phi}_1)$, and also define $\vec{S} \equiv (X, \vec{\phi}_2)$, where $\vec{\phi}_1$ and $\vec{\phi}_2$ are $N-1$ component vectors. Writing the superpotential (A.17) to cubic order, we have

$$W = m \Lambda X - \frac{1}{2} \frac{m}{\Lambda} X \vec{\phi}_1^2 + m \vec{\phi}_1 \cdot \vec{\phi}_2 + \dots \quad (\text{A.18})$$

The theory (A.17) now coincides with (A.14), with $n = N-1$, $m_i = \Lambda$, $h_i = -m/\Lambda$, and $f = m\Lambda$. Because all $y_i = |h_i f/m_i^2| = 1$, each component of the O’Raifeartaigh field $\vec{\phi}_1$ includes a real massless scalar. In the present model we identify them with the $\text{SO}(N)/\text{SO}(N-1)$ Goldstone bosons. The one-loop mass (A.16) is here

$$m_X^2 = (N-1) \frac{|m|^4}{16\pi^2 |\Lambda|^2} (\log 4 - 1). \quad (\text{A.19})$$

For $N = 6$, (A.17) is the effective macroscopic theory of the $\text{SU}(2)$ model, with $N_f = 2$ and $W = \lambda S^{ij} V_{ij}$, of [4, 5]. There $m = \lambda\Lambda$, with Λ the dynamical scale of the $\text{SU}(2)$ gauge theory, which also enters in the constraint (A.17) [12]. For this theory, essentially the above perturbative analysis, showing that the one loop potential of the effective theory pushes the pseudo-modulus to the origin, was given in [11].

A.4 Further generalizations

More generally, let us couple a field X to N fields ϕ_i via:

$$W = fX + \frac{1}{2} \phi_i M(X)^{ij} \phi_j. \quad (\text{A.20})$$

The example (A.4) has $M(X) = h \begin{pmatrix} X & m \\ m & 0 \end{pmatrix}$, linear in X , but more generally $M(X)$ need not be linear in X . Taking all fields to have canonical Kähler potential, the classical potential for the scalars is $V_{\text{tree}} = |F_X|^2 + F_{\phi_i}^\dagger F_{\phi_i}$, with

$$F_X = f + \frac{1}{2} \phi_i M'(X)^{ij} \phi_j, \quad F_{\phi_i} = M(X)^{ij} \phi_j. \quad (\text{A.21})$$

If $\det M(X)$ depends on X , then there will necessarily be values $X = X_0$ where it vanishes, and then $F_{\phi_i} = 0$ has a solution for non-vanishing ϕ_i^0 . In this case, there are generally supersymmetric vacua. These supersymmetric vacua could be endpoints of runaway directions. As a simple example with a runaway, consider $W = fX + \frac{1}{2}X^2\phi^2$, with $F_X = f^2 + X\phi^2$ and $F_\phi = X^2\phi$. The potential has a runaway, to a supersymmetric vacuum at $X = -f/\phi^2$, with $\phi \rightarrow 0$.

If $\det M(X)$ is a non-zero, X independent constant (as in the model (A.4)), then the only solution of $F_{\phi_i} = 0$ is $\phi_i = 0$. If $\det M(X)$ is a non-zero constant, but $M(X)^{ij}$ is not linear in X , then there is a possible runaway to a supersymmetric vacuum; one must check the particular model in more detail. If $\det M(X)$ is a non-zero constant, and $M(X)^{ij}$ is linear in X , then there is no runaway direction and supersymmetry is broken, generalizing the O’Raifeartaigh model, where $M(X) = h \begin{pmatrix} X & m \\ m & 0 \end{pmatrix}$. If $\det M(X)$ vanishes identically, then one must check further the particular model to determine whether or not supersymmetry is broken.

A.5 Comments about integrating out

Consider a theory with N chiral superfields ϕ_i and a superpotential

$$W = \frac{1}{2}\phi_i M^{ij} \phi_j + \text{terms involving other fields.} \tag{A.22}$$

We take M^{ij} to be a symmetric matrix of background superfields. The other fields can lead to M having a non-zero, supersymmetry breaking, F component, F_M .

We now integrate out ϕ_i . The result is a supersymmetric effective action for the background superfields M^{ij} . Because our theory is quadratic in ϕ_i , the effective Kähler potential for M^{ij} is exact at one-loop:

$$\begin{aligned} K_{\text{eff}} &= -\frac{1}{32\pi^2} \text{Tr} \left[M^\dagger M \log(M^\dagger M/\Lambda^2) \right] \\ &= -\frac{1}{2} \text{Tr} \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 + M^\dagger M} + \text{const.} \end{aligned} \tag{A.23}$$

Here the integrals are regulated in the UV by Λ and the constant is proportional to Λ^2 . This expression is familiar from the study of a theory with dynamical M , where it arises from the one loop renormalization of the kinetic term of M .

One way to see that (A.23) is correct is to expand it in components and focus on the term proportional to $F_M F_M^\dagger$:

$$\begin{aligned} \int d^4\theta K_{\text{eff}} \Big|_{F_M F_M^\dagger} &= -\frac{1}{2} \text{Tr} \int \frac{d^4p}{(2\pi)^4} \left(\Delta^{-2} M^\dagger F_M \Delta^{-1} F_M^\dagger M \right. \\ &\quad \left. + \Delta^{-2} F_M^\dagger M \Delta^{-1} M^\dagger F_M - \Delta^{-2} F_M^\dagger F_M \right) \\ &= -\frac{1}{2} \text{Tr} \int \frac{d^4p}{(2\pi)^4} \left(\tilde{\Delta}^{-2} (\tilde{\Delta} - p^2) F_M \Delta^{-1} F_M^\dagger \right. \\ &\quad \left. + \Delta^{-2} F_M^\dagger \tilde{\Delta}^{-1} (\tilde{\Delta} - p^2) F_M - \Delta^{-2} F_M^\dagger F_M \right) \end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{2} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \left(\tilde{\Delta}^{-1} F_M \Delta^{-1} F_M^\dagger + p^2 \frac{d}{dp^2} \Delta^{-1} F_M^\dagger \tilde{\Delta}^{-1} F_M \right) \\
&= +\frac{1}{2} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \tilde{\Delta}^{-1} F_M \Delta^{-1} F_M^\dagger
\end{aligned} \tag{A.24}$$

where $\Delta = p^2 + M^\dagger M$ and $\tilde{\Delta} = p^2 + M M^\dagger$. In the second line we used the fact that $M f(M^\dagger M) M^\dagger = f(M M^\dagger) M M^\dagger = M M^\dagger f(M M^\dagger)$ for every function f , and in the last line we have integrated by parts. The final result agrees with a one loop diagram with two external fields F_M and F_M^\dagger , and thus confirms our expression for (A.23).

The full effective action includes terms which are higher order in F_M and F_M^\dagger . Again, since the ϕ_i are free, they can be integrated out exactly at one-loop, and then the full effective action can be evaluated as a supertrace over the masses of the particles,

$$\mathcal{L}_{\text{eff}} = -\frac{1}{64\pi^2} \text{Str} \mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} = -\frac{1}{4} \text{Str} \int \frac{d^4 p}{(2\pi)^2} \frac{\mathcal{M}^2}{p^2 + \mathcal{M}^2}. \tag{A.25}$$

The bosonic mass-squared matrix for the fields $(\phi \ \phi^*)$ is $m_B^2 = E + H$ with

$$E \equiv \begin{pmatrix} M^\dagger M & 0 \\ 0 & M M^\dagger \end{pmatrix}, \quad H \equiv \begin{pmatrix} 0 & F_M^\dagger \\ F_M & 0 \end{pmatrix}, \tag{A.26}$$

where the components are for $(\phi \ \phi^*)$ and $(\phi^* \ \phi)^T$. The fermion mass-squared matrix is $m_F^2 = E$. If we expand (A.25) in powers of F_M and F_M^\dagger , the leading term coincides with that obtained from the effective Kähler potential (A.24); to show this we define $\Gamma \equiv p^2 + E$,

$$\begin{aligned}
\mathcal{L}_{\text{eff}} &= -\frac{1}{4} \text{Tr} \int \frac{d^4 p}{(2\pi)^2} \frac{m_B^2}{p^2 + m_B^2} + \frac{1}{4} \text{Tr} \int \frac{d^4 p}{(2\pi)^2} \frac{m_F^2}{p^2 + m_F^2} \\
&= -\frac{1}{4} \text{Tr} \int \frac{d^4 p}{(2\pi)^2} [(E + H)(1 + \Gamma^{-1} H)^{-1} - E] \Gamma^{-1} \\
&= -\frac{1}{8} \text{Tr} \int \frac{d^4 p}{(2\pi)^2} p^2 \frac{d}{dp^2} (H \Gamma^{-1})^2 + \mathcal{O}(F^4), \\
&= +\frac{1}{2} \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \tilde{\Delta}^{-1} F_M \Delta^{-1} F_M^\dagger + \mathcal{O}(F^4).
\end{aligned} \tag{A.27}$$

This agrees with the expression (A.24), coming from the effective Kähler potential (A.23). However, (A.23) does not capture the terms of higher order in F_M and F_M^\dagger in the first two lines of (A.27).

B. Calculating a and b

B.1 $SU(N)$ case

In this appendix, we flesh out the calculation of the one-loop effective potential (2.12) on the pseudo-moduli space of the $SU(N)$ macroscopic theory. As noted in section 2, this calculation reduces to determining two numerical coefficients a and b . More generally, the

one-loop potential is computed from the one-loop vacuum energy (1.4), treating the pseudo-moduli as a classical background. It thus suffices to expand away from the vacuum (2.7) along a two parameter space labelled by X_0 and θ :

$$\Phi = \begin{pmatrix} \delta Y & \delta Z^T \\ \delta \tilde{Z} & X_0 \mathbb{1}_{N_f - N} + \delta \widehat{\Phi} \end{pmatrix}, \quad \varphi = \begin{pmatrix} \mu e^\theta \mathbb{1}_N + \delta \chi \\ \delta \rho \end{pmatrix}, \quad \tilde{\varphi}^T = \begin{pmatrix} \mu e^{-\theta} \mathbb{1}_N + \delta \tilde{\chi} \\ \delta \tilde{\rho} \end{pmatrix}, \quad (\text{B.1})$$

with X_0 and θ treated as small parameters. To compute (1.4), we need the classical masses of the fluctuations in (B.1), as functions of the small pseudo-moduli background. This yields the one-loop correction to the vacuum energy,

$$\langle V_{\text{eff}}^{(1)} \rangle = \text{const.} + h^4 \mu^2 \left(\frac{1}{2} a N \mu^2 (\theta + \theta^*)^2 + b (N_f - N) |X_0|^2 \right) + \dots, \quad (\text{B.2})$$

from which we can read off the coefficients a and b . (For simplicity we take h and μ real and positive throughout this appendix.)

To compute the classical masses, we substitute (B.1) into the superpotential (2.4):

$$\begin{aligned} W &= h \text{Tr} \varphi \Phi \tilde{\varphi} - h \mu^2 \text{Tr} \Phi \\ &= h \text{Tr} \left[\mu e^\theta \delta Z^T \delta \tilde{\rho} + \mu e^{-\theta} \delta \tilde{Z}^T \delta \rho + \delta \rho^T (X_0 + \delta \widehat{\Phi}) \delta \tilde{\rho} - \mu^2 (X_0 + \delta \widehat{\Phi}) \right. \\ &\quad \left. + \mu e^\theta \delta Y \delta \tilde{\chi} + \mu e^{-\theta} \delta Y^T \delta \chi \right] + \dots \end{aligned} \quad (\text{B.3})$$

where \dots contains terms of cubic order and higher in the fluctuations. According to (B.3), the off-diagonal components of $\delta \widehat{\Phi}$ do not contribute to the mass matrix, so we can neglect them here. Moreover, the fields $\delta \chi$, $\delta \tilde{\chi}$, and δY only couple to the supersymmetry breaking fields $\delta \rho$ and $\delta \tilde{\rho}$ through terms of cubic or higher order in the fluctuations. Therefore, the mass matrix for these fields will be supersymmetric, and they will not contribute to the supertrace. So they can also be neglected here. The remaining relevant terms are

$$W \supset h \sum_{f=1}^{N_f - N} \left[(X_0 + \delta \widehat{\Phi}_{ff}) (\delta \rho \delta \tilde{\rho}^T)_{ff} + \mu e^\theta (\delta \tilde{\rho} \delta Z^T)_{ff} + \mu e^{-\theta} (\delta \rho \delta \tilde{Z}^T)_{ff} - \mu^2 (X_0 + \delta \widehat{\Phi}_{ff}) \right]. \quad (\text{B.4})$$

We recognize $N_f - N$ decoupled copies of an O’Raifeartaigh-like model of the form

$$W = h \left(X \vec{\phi}_1 \cdot \vec{\phi}_2 + \mu e^{-\theta} \vec{\phi}_1 \cdot \vec{\phi}_3 + \mu e^\theta \vec{\phi}_2 \cdot \vec{\phi}_4 - \mu^2 X \right) \quad (\text{B.5})$$

where the $\vec{\phi}_i$ are N dimensional vectors. A calculation completely analogous to those in appendix A yields the one-loop vacuum energy coming from these $N_f - N$ O’Raifeartaigh-like models, as a function of $\langle X \rangle = X_0$ and θ . We find:

$$\langle V_{\text{eff}}^{(1)} \rangle = \text{const.} + \frac{h^4 \mu^2 (\log 4 - 1) N (N_f - N)}{8\pi^2} \left(\frac{1}{2} \mu^2 (\theta + \theta^*)^2 + |X|^2 \right) + \dots \quad (\text{B.6})$$

Comparing with (B.2), we read off the coefficients a and b :

$$a = \frac{\log 4 - 1}{8\pi^2} (N_f - N), \quad b = \frac{\log 4 - 1}{8\pi^2} N \quad (\text{B.7})$$

This is the answer (2.13) quoted in section 2.

B.2 SO(N) case

The SO(N) macroscopic model studied in section 6 can also be analyzed along the lines of the previous subsection. To begin, we expand around a point near (6.6),

$$\Phi_0 = X_0 \mathbb{1}_{N_f - N}, \quad \varphi_0 = \mu \begin{pmatrix} \cosh \theta & i \sinh \theta \\ -i \sinh \theta & \cosh \theta \end{pmatrix} \otimes \mathbb{1}_{N/2} \quad (\text{B.8})$$

where for simplicity we are assuming N is even. The general form of the one-loop vacuum energy, expanded around $X_0 = \theta = 0$, is

$$\langle V_{\text{eff}}^{(1)} \rangle = \text{const.} + h^4 \mu^2 \left(\frac{1}{8} a N \mu^2 (\theta + \theta^*)^2 + b (N_f - N) |X_0|^2 \right) + \dots \quad (\text{B.9})$$

To calculate the coefficients a and b , we reduce the superpotential as in the previous subsection, yielding the relevant terms

$$W \supset h \sum_{f=1}^{N_f - N} \left[(X_0 + \delta \widehat{\Phi}_{ff}) (\delta \rho \delta \rho^T)_{ff} + \sqrt{2} (\delta \rho \varphi_0^T \delta Z^T)_{ff} - \mu^2 (X_0 + \delta \widehat{\Phi}_{ff}) \right] \quad (\text{B.10})$$

This is equivalent to $N_f - N$ decoupled copies of the O’Raifeartaigh-like model

$$W = h \left[X (\vec{\phi}_1^2 + \vec{\phi}_2^2) + \sqrt{2} \mu \begin{pmatrix} \vec{\phi}_1 \\ \vec{\phi}_2 \end{pmatrix}^T \begin{pmatrix} \cosh \theta & -i \sinh \theta \\ i \sinh \theta & \cosh \theta \end{pmatrix} \begin{pmatrix} \vec{\phi}_3 \\ \vec{\phi}_4 \end{pmatrix} - \mu^2 X \right] \quad (\text{B.11})$$

where the $\vec{\phi}_i$ are $N/2$ dimensional vectors. By a unitary transformation,

$$(\vec{\phi}_1, \vec{\phi}_2, \vec{\phi}_3, \vec{\phi}_4) \rightarrow \left(-\frac{i(\vec{\phi}_1 - \vec{\phi}_2)}{\sqrt{2}}, \frac{\vec{\phi}_1 + \vec{\phi}_2}{\sqrt{2}}, \frac{i(\vec{\phi}_3 - \vec{\phi}_4)}{\sqrt{2}}, \frac{\vec{\phi}_3 + \vec{\phi}_4}{\sqrt{2}} \right) \quad (\text{B.12})$$

we can actually turn (B.11) into

$$W = h \left(2X \vec{\phi}_1 \cdot \vec{\phi}_2 + \sqrt{2} \mu e^{-\theta} \vec{\phi}_1 \cdot \vec{\phi}_3 + \sqrt{2} \mu e^{\theta} \vec{\phi}_2 \cdot \vec{\phi}_4 - \mu^2 X \right) \quad (\text{B.13})$$

which is the O’Raifeartaigh-like model of the previous subsection, but with $\mu_{\text{here}} = \sqrt{2} \mu_{\text{there}}$ and $h_{\text{here}} = \frac{1}{2} h_{\text{there}}$. Therefore, we can copy over the vacuum energy from the previous subsection, rescaled appropriately:

$$\langle V_{\text{eff}}^{(1)} \rangle = \text{const.} + \frac{h^4 \mu^2 (\log 4 - 1) N (N_f - N)}{2\pi^2} \left(\frac{1}{4} \mu^2 (\theta + \theta^*)^2 + |X|^2 \right) + \dots \quad (\text{B.14})$$

Comparing with (B.9), we can read off a and b . The result is the answer (6.10) quoted in section 6.

B.3 Sp(N) case

Finally, let us analyze the Sp(N) macroscopic model of section 6 in the same way. We expand around a point near (6.18),

$$\Phi_0 = X_0 \mathbb{1}_{N_f - N} \otimes (i\sigma_2), \quad \varphi_0 = \mu \begin{pmatrix} \cosh \theta & i \sinh \theta \\ -i \sinh \theta & \cosh \theta \end{pmatrix} \otimes \mathbb{1}_N \quad (\text{B.15})$$

(Recall our conventions are such that $J_{2N} = \mathbb{1}_N \otimes (i\sigma_2)$.) The general form of the one-loop vacuum energy, expanded around $X_0 = \theta = 0$, is

$$\langle V_{\text{eff}}^{(1)} \rangle = h^4 \mu^2 \left(\frac{1}{2} 2Na\mu^2 (\theta + \theta^*)^2 + 2(N_f - N)b|X|^2 \right) + \dots \quad (\text{B.16})$$

To calculate (B.16), we again expand the superpotential and reduce it as in the previous subsections. This yields precisely the same O’Raifeartaigh model (B.11) as for $\text{SO}(N)$, except with $(N, N_f - N)$ in $\text{SO}(N)$ replaced with $(4N, N_f - N)$. Therefore, the one-loop vacuum energy is just (B.14) multiplied by four,

$$\langle V_{\text{eff}}^{(1)} \rangle = \text{const.} + \frac{2h^4 \mu^2 (\log 4 - 1) N (N_f - N)}{\pi^2} \left(\frac{1}{4} \mu^2 (\theta + \theta^*)^2 + |X|^2 \right) + \dots \quad (\text{B.17})$$

Comparing with the general form (B.16) and reading off a and b , we obtain the answer (6.23) quoted in the text.

C. A landscape of supersymmetry breaking vacua

Consider $\mathcal{N} = 1$ supersymmetric SQCD, with gauge group $\text{SU}(N_c)$ and N_f flavors, and add an extra chiral superfield Φ in the adjoint representation, with superpotential (see e.g. [23, 24]–[25])

$$W = \sum_{p=1}^{K+1} \frac{1}{p} \text{Tr } g_p \Phi^p + \text{Tr } m M. \quad (\text{C.1})$$

(For simplicity we do not include superpotential terms coupling Φ to the fundamentals. They can be easily added.) Let us consider the case of large g_p , where we should expand around the classical vacua of (C.1). There is a “landscape” of such classical vacua, with $\text{SU}(N_c)$ Higgsed by the $\langle \Phi \rangle$ as

$$\text{U}(N_c) \rightarrow \prod_{i=1}^K \text{U}(N_i) \quad \text{for all partitions} \quad N_c = \sum_{i=1}^K N_i; \quad N_i \geq 0. \quad (\text{C.2})$$

The number of such possibilities grows rapidly with K and N_c .

For generic and large g_p , all of the components of Φ in each of these vacua are massive. The low-energy theory in each vacuum consists of approximately decoupled $\text{U}(N_i)$ gauge groups. Each $\text{U}(N_i)$ group has N_f flavors, with identical masses given by m in (C.1). Suppose now that at least one N_i satisfies

$$N_i + 1 \leq N_f < \frac{3}{2} N_i \quad (\text{C.3})$$

then, using the analysis in sections 2 — 5, the $\text{U}(N_i)$ theory has meta-stable supersymmetry breaking vacua. We see that this theory has many supersymmetric as well as many compact spaces of meta-stable vacua. There is thus a landscape of supersymmetric and meta-stable non-supersymmetric vacua.

Such vacua are also present in the string theory landscape, as these gauge theories have string realizations. In this context the integers N_i arise as the number of branes or the values of certain fluxes.

As an aside, we note that one can also construct field theory examples with a landscape of non-supersymmetric vacua, with no supersymmetric vacuum. Consider, for example the supersymmetry breaking model of [3], based on $SU(N_c)$ gauge theory, with N_c odd, and matter in the $\square \oplus (N_c - 4)\bar{\square}$. As noted in [29, 30], it is interesting to consider adding to this theory an adjoint Φ , with superpotential as in (C.1). We again get a classical landscape of vacua for Φ , with the breaking patterns (C.2). In some of these vacua, the low-energy theory reduces to one that was already known to break supersymmetry [29, 30]. A priori, one might expect that some of the vacua break supersymmetry, and others might not. A systematic analysis has not yet been completed, but it seems possible that every vacuum of the classical landscape of (C.2) breaks supersymmetry in this present case.

D. $\mathcal{N} = 2$ super Yang-Mills, slightly broken to $\mathcal{N} = 1$

In $\mathcal{N} = 2$ supersymmetric gauge theory, the exact Kähler potential of the low-energy effective theory on the Coulomb branch can be determined, from a holomorphic quantity (the prepotential) [15]. Let us consider an $\mathcal{N} = 2$ theory, broken to $\mathcal{N} = 1$ by superpotential terms,

$$\Delta W_{\text{tree}} = \sum_p \frac{1}{p} g_p \text{Tr} \Phi^p \equiv \sum_p g_p u_p. \quad (\text{D.1})$$

The supersymmetric vacua of this theory have been much studied (see e.g. [31, 32]- [33]). We can also look for meta-stable minima of the effective potential on the Coulomb branch,

$$V_{\text{eff}} = \sum_{p\bar{p}} (K_{\text{eff}}^{-1})^{u_p u_{\bar{p}}^\dagger} g_p g_{\bar{p}}^*. \quad (\text{D.2})$$

Taking all $g_p \ll 1$, where $\mathcal{N} = 2$ is just slightly broken to $\mathcal{N} = 1$, we can use the exactly determined $\mathcal{N} = 2$ Kähler potential $K_{\text{eff}}(u_p, u_p^\dagger, \Lambda)$ in (D.2), to get the effective potential to leading order in g_p , but exactly in Λ . We can there look for meta-stable vacua, without the ambiguity of the order one coefficients α and β that appeared in section 5.

For example, consider $\mathcal{N} = 2$ supersymmetric $SU(2)$ Yang-Mills theory, broken to $\mathcal{N} = 1$ as in (D.1) by a mass term $g_2 = m_\Phi$. For $g_2 = 0$, the low-energy effective theory is an $\mathcal{N} = 2$ $U(1)$ vector multiplet. There is a moduli space of $\mathcal{N} = 2$ supersymmetric vacua, with Kähler metric given by [15]

$$ds^2 = \text{Im}\tau |da|^2, \quad \tau = \frac{da_D/du}{da/du}, \quad (\text{D.3})$$

with

$$a(u) = \frac{\sqrt{2}}{\pi} \int_{-1}^1 \frac{dx \sqrt{x-u}}{\sqrt{x^2-1}}, \quad \text{and} \quad a_D = \frac{\sqrt{2}}{\pi} \int_1^u \frac{dx \sqrt{x-u}}{\sqrt{x^2-1}}. \quad (\text{D.4})$$

The functions $a(u)$ and $a_D(u)$ can be expressed in terms of hypergeometric functions. The dynamical scale Λ was set to unity; it can be restored by dimensional analysis. Adding

$W_{\text{tree}} = m_\Phi u$ leads to supersymmetric vacua at $u = \pm 1$, where a massless monopole or dyon condenses [15]. We here ask if there could also be meta-stable, non-supersymmetric vacua, at other values of u . In this case, it turns out that the answer is no.

For small m_Φ , the scalar potential is

$$V_{\text{eff}}(u) = (\text{Im } \tau(u))^{-1} \left| \frac{da}{du} \right|^{-2} |m_\Phi|^2. \quad (\text{D.5})$$

it is straightforward to find that the only minima are the global ones, at $u = \pm 1$. There is a saddle point at $u = 0$, where the potential curves up along the $\text{Im } u$ axis, but down along the $\text{Re } u$ axis. The vacuum at $u = 0$ is unstable to rolling along the $\text{Re } u$ axis, down to the minima at $u = \pm 1$.

More generally, one could look for meta-stable non-supersymmetric vacua in $\mathcal{N} = 2$ supersymmetric $\text{SU}(N_c)$ SQCD, with N_f massive flavors, slightly broken to $\mathcal{N} = 1$ by (D.1). For $g_p = 0$, the effective theory of the Coulomb branch, and in particular the Kähler potential, are exactly given by the curve $y^2 = \det(x - \Phi)^2 - \Lambda^{2N_c - N_f} \prod_{f=1}^{N_f} (x + m_i)$ [34–38], where m_i are the masses of the flavors. Taking $g_2 = m_\Phi$ to infinity, the low-energy theory at $\Phi = 0$ is governed by $\mathcal{N} = 1$ SQCD. There, as we have argued, there are meta-stable, supersymmetry breaking vacua for $N_f < \frac{3}{2}N_c$. Perhaps the meta-stable vacua can also be seen in the opposite limit, where the $\mathcal{N} = 2$ breaking terms (D.1) are small, and the infrared theory can be approximately described using the exactly known $\mathcal{N} = 2$ supersymmetric Kähler potential.

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